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On the Design of a 15 Mev β-Ray Spectrometer*

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ABSTRACT. The design of a lens spectrometer with respect to economy in power consumption and copper weight, resolving and collecting power has been studied by small scale experiments. The construction and performance of a large β-ray spectrometer is described which focuses 15 Mev electrons when its coil is energized by 50 amp at 360 volts. The coil is cooled by a flow of 12 gallons of water per minute. Maximum resolution and collecting power of the spectrometer are 0·8% and 0·6% respectively. The influences of spherical aberration, source and window size on the resolution of the thick lens spectrometer are discussed.

§ 1. INTRODUCTION

SINCE the publication of the first account of a β-ray lens spectrometer (Klemperer 1935) a great number of these instruments have been described in the literature and a comprehensive survey of the various types of lens spectrometers has been given by Persico and Geoffrion (1950). Very little, however, seems to be known about the design of lens spectrometers for highest β-ray energies (10 Mev). Apart from an early instrument due to Cosslett (1940 a), a lens spectrometer for this highest β-ray energy region has been built by Hornyak and Lauritsen (1949, 1950), but no details about this latter instrument have been published. The high energy β-ray lens spectrometer which will be described here has proved its practical usefulness for the investigation of some high energy β-spectra during the last few years in the Imperial College, South Kensington; its main features have been given by Graham (1949). Attention has been given to the resolving and collecting power of the instrument, to the weight of copper wire and to the consumption of electric power. Also easy calibration at high energies and easy accessibility to the source and to the collector were desirable: hence the use of an iron shield was ruled out.

§ 2. EXPERIMENTS WITH A SMALL SCALE MODEL

The influence of the shape of a coil on its focusing properties has been studied here by means of tracing experiments with relatively slow electrons (0·5 to 5 kev) according to the method developed by Klemperer and Wright (1939). From these experiments a set of curves was obtained which made it possible to predict the focusing properties of any unshielded coil in symmetrical position with respect to object and image, given the object to image distance, the coil length and its inside and outside radii.

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* Substantially a part of the Ph.D. thesis by R. L. Graham (1949) with some further developments.
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The experimental arrangement is shown in fig. 1. Electrons leaving a hairpin filament A were accelerated through a potential V to the 'pepperpot' diaphragm D. The position or the potential of a grid-electrode B was adjusted for obtaining a practically point-like and aberration-free electron source in the centre of its aperture (cf. Klemperer and Klinger 1951). The diaphragm D contained rings of fine holes of 0·1 mm diameter in circles of radii 1, 2, 3 and 4 mm. This allowed a series of fine electron pencils having different values of aperture angle to enter the vacuum chamber E. The pencils were focused by the magnetic coil G on to the transparent fluorescent target F, which is in a fixed position and terminates the vacuum tube. Three single-layer coils G, of 65, 97 and 129 mm radius, were examined and tapped at 30 mm intervals along their lengths. As the shortest object-image distance z_f for any lens system occurs for the symmetrical case, the coils were mounted midway between the electron source and the fluorescent target, with $z_f = 632$ mm in the present experiments.

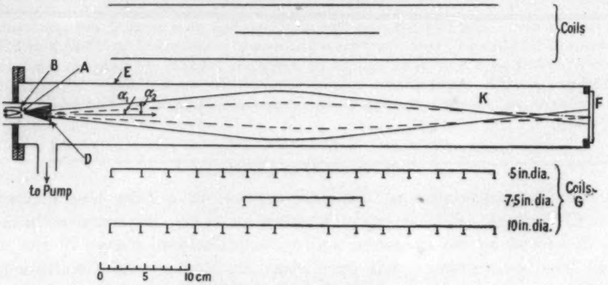

Fig. 1. Small scale model.

§ 3. SPHERICAL ABERRATION

The spherical aberration of the focusing coil is easily detected and measured by the change in coil current for focusing paraxial and marginal rays respectively. The relative change may be written as

$$\Delta I/I_0 = (I_0 - I_\alpha)/I_0, \qquad \ldots\ldots(1)$$

where I_0 is the focusing current for paraxial electrons and I_α the current required to bring into focus a marginal ray of the initial semi-aperture angle α. Usually, however, the spherical aberration of a lens is recorded by the coefficient

$$C_s = \Delta J/\alpha^2 = (J_0 - J_\alpha)/\alpha^2 \qquad \ldots\ldots(2)$$

where ΔJ is the longitudinal spherical aberration, i.e. J_0 and J_α are the object positions for a sharp image on the fixed target to be produced respectively by paraxial rays and by marginal rays of a semi-aperture α. A dimensionless aberration coefficient is frequently defined by C_s/f, where f is the focal length of the system. A focal length is, however, not easily measured if object and image are inside the field of a long coil. Moreover, no finite focal length would exist for a telescopic system like the homogeneous field of a solenoid. Hence it is reasonable to define for the present purpose a dimensionless aberration coefficient

$$\frac{C_s}{z_f}=\frac{J_0-J_\alpha}{\alpha^2 z_f}=\frac{-\Delta z_f}{\alpha^2 z_f}, \qquad \ldots\ldots(3)$$

where z_f is the object to image distance for symmetrical position of object and image, and for short coils $z_f=4f$. Now, for the projection of paraxial electrons of a given momentum by a given coil, z_f is inversely proportional to the *m*th power of the coil current, i.e.

$$z_f=k/I_0^m, \qquad \ldots\ldots(4)$$

where *k* is a constant and the exponent *m* is a function of the coil length. *m* can take any value between the limits 2 and 1, being 2 for the short coil and 1 for the solenoid.

Differentiation of eqn. (4) yields

$$\Delta z_f=-mz_f\Delta I/I_0 \qquad \ldots\ldots(5)$$

and substitution of z_f from eqn. (5) into eqn. (3) leads to

$$\frac{C_s}{z_f}=\frac{m\Delta I}{\alpha^2 I_0}=\frac{m(I_0-I_\alpha)}{\alpha^2 I_0}. \qquad \ldots\ldots(6)$$

According to eqn. (6) $\Delta I/I_0$ is proportional to α^2, and for a given coil depends on α only. This has been found to be in agreement with the facts for the small angles α used here. In the absence of a magnetic field, the small semi-aperture angles α could be determined as the ratios of the radii of circles of holes in the pepperpot diaphragm D to the distance BD from grid aperture to diaphragm. However, in the presence of a magnetic field between B and D as produced by the coils G, all angles α were estimated to be approximately 17% smaller than the above ratio, a sinusoidal path of the electrons in the magnetic field being assumed.

The factor *m* can be determined, as will be shown in § 7, from the experimental curves for the focusing efficiency given in fig. 8. However, for the purpose of comparing coils of different dimensions it is sufficient to investigate the coefficient $(I_0-I_\alpha)/\alpha^2 I_0=C_s/mz_f$. Experimental values for this aberration coefficient obtained from measurements with $\alpha=0{\cdot}1$ radian are shown in fig. 2. The aberration increases rapidly with decreasing coil radius *a* and with decreasing coil length *L*. The aberration coefficient of the homogeneous field is indicated by a dotted line. There the value of C_s/z_f is given by

$$\Delta z_f=z_f(1-\cos\alpha)\simeq z_f(\alpha^2/2), \qquad \ldots\ldots(7)$$

from which, with $m=1$ and with eqns. (3) and (6),

$$C_s/z_f=(I_0-I_\alpha)/\alpha^2 I_0=\tfrac{1}{2}. \qquad \ldots\ldots(8)$$

Towards this value all curves of fig. 2 converge for $L/z_f \gg 1$.

Little information is given in the literature about the spherical aberration of electron lenses when object and image are in symmetrical position. The available experimental data generally refer to parallel incidence where the aberration is known to be smallest. For this case, for instance, Becker and Wallraff (1938, 1940) measured $C_s/f=3$ for an air coil of length $L=0{\cdot}26a$, where *a* is the coil radius. The aberration coefficient was found to decrease with increasing coil length according to the law

$$C_s/f \propto L^{-0{\cdot}4}. \qquad \ldots\ldots(9)$$

The values of C_s derived from the results for relatively short coils as presented in fig. 2 are seen to be much larger and to increase much more rapidly with decreasing coil length than the C_s values for parallel incidence. According to the data of fig. 2, the difference between aberration coefficients for parallel and divergent incidence increases as the ratio of coil radius a to object–image distance z_f decreases. Hence the conclusion may be drawn that a reduction of aberration can be obtained by spreading the magnetic field beyond the interval between object and image. However, this spreading implies a waste of electrical energy, since this energy is partially used for building up an ultra-focal field which is not utilized for focusing the electrons.

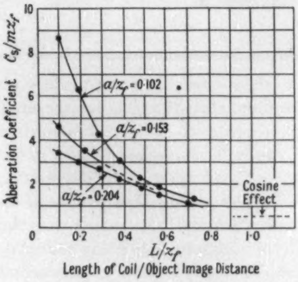

Fig. 2. Dimensionless spherical aberration coefficients for symmetrically placed single-layer coils of different radii *a*.

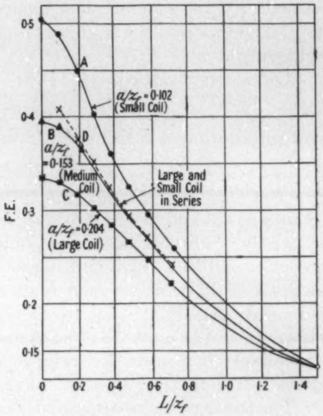

Fig. 3. Focusing efficiency for paraxial electrons.

§ 4. FOCUSING EFFICIENCY

The arrangement shown in fig. 1 was used to find the best coil according to the criteria outlined above.

For convenience let the following term F.E. be called the 'focusing efficiency' of the coil,

$$\text{F.E.} = \frac{r_e B_e}{nI\mu}, \qquad \ldots\ldots(10)$$

where $r_e B_e = p/\boldsymbol{e}$, the momentum divided by electron charge is measured, as usual, by the radius of curvature r_e (cm) of the orbit in the field of flux density B_e (gauss); nI is the number of ampere turns of the coil and $\mu = 1$ is the magnetic permeability.

Focusing efficiency data were measured with the arrangement shown in fig. 1 by maintaining the accelerating potential at 1 kv and measuring the coil currents I_α which were necessary to focus electron pencils of a given semi-aperture α on to a target in symmetrical position to the source. From a set of I_α values the current I_0 for focusing paraxial rays ($\alpha \rightarrow 0$) was extrapolated (cf. § 3). Focusing efficiencies as a function of the coil length L over object–image distance z_f are presented in fig. 3. The full curves A, B, C were taken with the three single-layer coils shown in fig. 1. The broken curve D represents the focusing efficiencies measured when corresponding sections of the large and small coil were connected in series. The interesting result is derived that within the limits of experimental error ($\pm 2\%$) each point of the curve for the compound coil represents the arithmetic mean of the focusing efficiencies determined for each coil separately. The focusing efficiency (curve D) is distinctly greater than that of the medium coil (curve B), the diameter of which is half way between the diameters of the larger and the smaller coil.

To supplement the experimental data calculations were made, both for a very long coil and for a current loop. In solenoids longer than the object–image distance ($L \geqslant z_f$) the magnetic field is nearly uniform and of flux density

$$B_0 = 0{\cdot}4\,\pi n I \mu / L, \qquad \ldots\ldots(11)$$

while the object–image distance is given by

$$z_f = 2\pi(r_e B_e)/B_0. \qquad \ldots\ldots(12)$$

Hence from eqns. (10), (11) and (12) follows

$$\text{F.E.} = 0{\cdot}2\, z_f/L. \qquad \ldots\ldots(13)$$

The focusing efficiency of the current loop can be calculated by substitution of its axial flux density distribution given by

$$B_z = \frac{2\pi\mu I}{a}\left(\frac{1}{1+(z/a)^2}\right)^{3/2}, \qquad \ldots\ldots(14)$$

into Busch's well-known lens formula, with the result

$$\text{F.E.} = 0{\cdot}171(z_f/a)^{1/2}. \qquad \ldots\ldots(15)$$

It is known, however (cf. Siday 1942), that Busch's formula leads to correct results only so long as the object–image distance z_f is large in comparison with the 'thickness' of the electron lens.

The actual focusing efficiency can be obtained from the paraxial ray equation

$$\frac{d^2r}{dz^2} = -\frac{B_z^2 r^2}{(2B_e r_e)^2}, \qquad \ldots\ldots(16)$$

which, upon substitution of B_z from eqn. (14), can be integrated by analytical methods. The procedure and the solution are complicated, and we refer for details to the thesis by Graham (1949). Some numerical results for the focusing efficiency of the current loop thus calculated are presented in fig. 4 (full curve); the actual values are seen to be distinctly smaller than those given by the Busch approximation (broken curve).

For convenience, the data of fig. 3 are replotted in fig. 5, which gives curves showing the dependence of focusing efficiency upon the coil radius for a given coil length. The efficiency drops off with increasing coil radius. With the help of fig. 5 it is possible to predict the focusing efficiency of any multi-layer coil of length L, of inside and outside radii a_1 and a_2 and with given object–image distance z_f. According to the above rule about averaging the focusing efficiencies of single layers, the focusing efficiency of the coil is found from the curve of fig. 5 with the corresponding L/z_f at the average height between a_1/z and a_2/z_f.

§ 5. DETERMINATION OF THE OPTIMUM DIMENSIONS FOR THE FOCUSING COIL

In the design of high duty focusing coils allowance has to be made for water cooling between sections of the windings. With a minimum of electrical insulation it is then possible to dissipate the heat from wires carrying about 300 amp/cm^2; 25–30 % of the winding cross section has to be reserved for cooling and insulation. Equation (10) and figs. 2 and 5 provide the data to predict the dimensions and properties of a wide range of coils.

Results of a number of calculations have been plotted in fig. 6 (*a*). There the outer radius a_2 is plotted against the length L of a coil which focuses 15 Mev electrons in a distance $z_f = 100$ cm at a fixed inner coil radius $a_1 = 10$ cm. From the constructional point of view the longer coils with relatively smaller outer radius seem to be preferable; they would be more easily handled in the workshop.

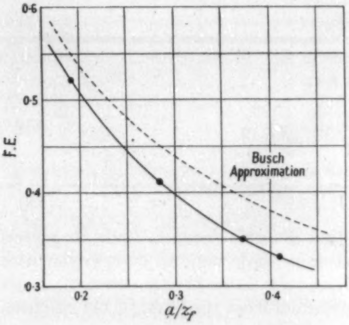

Fig. 4. Focusing efficiency for current loop.

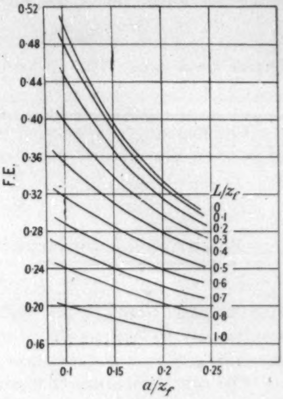

Fig. 5. Focusing efficiency and coil radius.

The collecting power of the lens spectrometer depends upon the ratio of the inner coil radius a_1 to the object–image distance z_f. In fig. 6 (*b*) this ratio is plotted as a function of z_f for 15 Mev electrons, for an outer coil radius $a_2 = 23$ cm, a mean current density in the windings of 230 amp/cm^2 and a coil length L equal to 80 % of the object–image distance z_f. The curve in fig. 6 (*b*) has a flat maximum near $z_f = 100$ cm, and a value of this magnitude is therefore adopted for the source to collector distance in the spectrometer.

The mass of copper required to focus 15 Mev electrons with the coil described by fig. 6 (*a*) is estimated for various lengths L of the coil, assuming that 75 % of the winding volume is occupied by solid copper. The results plotted in fig. 6 (*c*) show a minimum for $L = 55$ cm.

Finally, in fig. 6 (*d*) there are plotted the aberration coefficients $(I_0 - I_\alpha)/\alpha^2 I_0$ estimated for the above range of coil dimensions under the assumption that they are the same as for a single layer having a diameter equal to the mean diameter of the coil. The values are estimated from fig. 2. The sphericaaberration is seen to decrease with increasing coil length, indicating the desirability of having as long a coil as possible. Long coils bend the electrons gradually over their whole length. Electrons of a given emission angle travel the nearer to the axis the longer the coils, and for a given inside coil radius longer coils have greater collecting power.

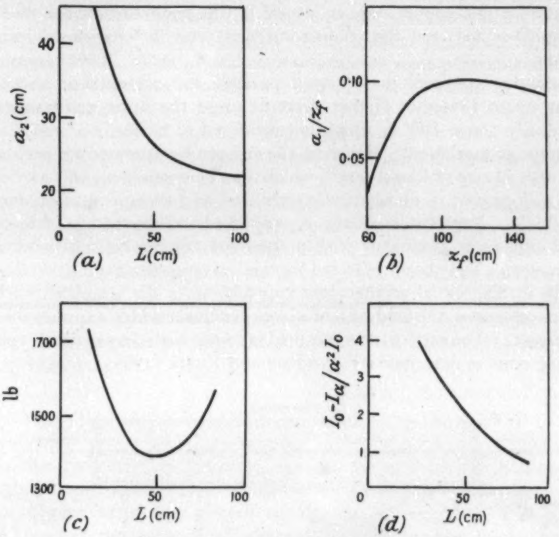

Fig. 6. Properties of a 15 MeV spectrometer.

(*a*) Outer coil radius versus length. (Inner coil radius and source to collector distance are fixed by a required collecting power.)

(*b*) Inner coil radius versus source to collector distance. (Given: outer coil radius and ratio (80%) of coil length to source to collector distance.)

(*c*) Mass of copper versus coil length.

(*d*) Spherical aberration coefficient of the multilayer coil versus the coil length.

§ 6. THE HIGH ENERGY LENS SPECTROMETER

(i) *The Focusing Coil*

As shown in fig. 7, the focusing coil of the spectrometer consists of two identical halves which, upon assembly, are bolted together. Accurate alignment is ensured by dowels. The overall dimensions of the assembled coil are: inside diameter 18·8 cm, outside diameter 46·6 cm, length 90 cm including the 4 cm thick end flanges, which contain all hydraulic and electrical connections. The whole coil which has a weight of about 15 cwt is mounted horizontally in a specially designed cradle. The windings consist of a total of 4680 turns of paper bonded enamel-insulated copper strip of 0·1 in. × 0·2 in. cross section with a total resistance of 7 ohms.

As can be seen from fig. 7, two sections of layers of windings of the coil are sandwiched between three cooling layers which consist respectively of (i) flattened copper tubing cast in lead, (ii) a hollow brass cylinder in which the water is conducted along a spiral groove and which is slipped over the inner section of windings, and (iii) composition tubing soldered to copper sheet wrapped tightly around the outermost windings. The cooling water passes these three cooling bodies in parallel. When the coils are operated at full power, twelve gallons of water per minute are needed at a pressure of 50 lb/in^2. To monitor the coil temperature a fine copper wire is wound in the grooves between the turns of layers, midway between the cooling surfaces, and its change in resistance is observed.

Continuous operation has proved possible for currents of about 40 amp (focusing up to 13 MeV). Higher currents cause the measured temperature to rise gradually above 100° C, which is considered to be the maximum safe limit for this type of insulation. However, the coil can be operated for periods of half an hour with 52 amp (4·0 amp/mm^2), which is sufficient to focus 16·7 MeV electrons.

The coil current is automatically stabilized and measured to an accuracy of within 0·1%. Negative feedback is employed. The voltage drop across a standard resistor is compared with a standard voltage by means of a mirror galvanometer; a light beam reflected by this galvanometer and operating on two photocells in differential arrangement controls, via a d.c. amplifier and a small amplidyne generator, the field coils of a large generator which supplies the spectrometer current. The earth's magnetic field is compensated by bent up, rectangular degaussing coils as described by Lyddane and Ruark (1939).

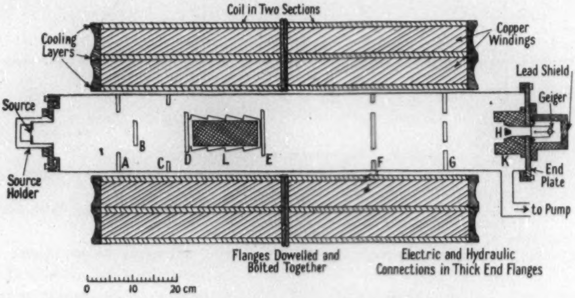

Fig. 7. High energy β-ray lens spectrometer.

(ii) *The Vacuum Chamber*

As shown in fig. 7, the vacuum chamber consists of a copper tube 17 cm in diameter, 104 cm long, with end plates bolted to flanges and made vacuum-tight by rubber rings. The end plates contain the source holder and the lead-shielded Geiger counter respectively. The vacuum chamber is adjusted to the axis of the focusing coil by means of alignment screws. The inside of the tube is lined with aluminium sheet to reduce scattering from the walls. The path of the rays is defined by seven baffles A, B, C, D, E, F and G. They consist of ¼ in. aluminium rings and discs respectively; the discs are either supported by three equidistant thin rods converging from the tube wall towards the axis, or connected to one another by a long axial rod. The central baffle L is shaped as shown in the figure and contains a lead cylinder for reducing the effect of direct γ-radiation. The edges of all disc and ring baffles are bevelled to avoid glancing reflection of β-rays into the collector aperture; for the same reason the aluminium sheath around the lead baffle L is serrated.

The location of the baffles was determined after tracing the shape of the electron trajectories. For this purpose the magnetic flux density along the axis was measured with a search coil and the electron path was obtained by numerical integration of the paraxial ray equation in the manner described by Goddard and Klemperer (1944). This procedure is satisfactory since paraxial and marginal trajectories are not significantly different and since the location of the defining baffles is not critical.

Two different kinds of collector apertures have been used. In the first arrangement a circular collector aperture was placed directly in front of the thin window of the Geiger counter at a distance of 106 cm from the source. In the second arrangement a ring selector diaphragm was inserted at the ring cross-over of the rays 4·2 cm in front of the Geiger counter window. The outer boundary of the annular slit aperture in this diaphragm is formed by the inner wall of a hollow lead cylinder K, shaped as shown in fig. 7. The inner boundary of the aperture is formed by the truncated lead cone H which, resting on three fine radial screws, can be shifted in axial direction in order to vary the effective width of the slit. The distance between the source and the plane of the ring selector aperture is 103 cm.

§7. THE PERFORMANCE OF THE SPECTROMETER

The collecting power and the resolution of the spectrometer were of primary interest. The resolution is obtained experimentally from the half-maximum width of the 'transmission curve', which is the curve representing the counting rate for monochromatic rays plotted against the coil current. The collecting power is given as the ratio of all electrons received at the maximum of the transmission curve to the total emission from the source. For small source dimensions the collecting power Ω can be determined geometrically from the minimum and maximum admitted semi-apertures α_1 and α_2 respectively:

$$\Omega = \tfrac{1}{2}(\cos\alpha_2 - \cos\alpha_1). \qquad \ldots\ldots(17)$$

Figures 8 and 9 show transmission curves obtained with the spectrometer using circular apertures of different diameter W and for different source diameters S. Both curves are taken with β-rays of the thorium F-line (148·3 kev or 1388·6 gauss cm). The collecting power is $\Omega = 0{\cdot}5\,\%$ of 4π as calculated from eqn. (17), the semi-apertures ranging in both cases from 7° 44′ to 11° 9′.

For fig. 8 the source is an aluminium foil of about 1 mg/cm² mass thickness uniformly covered with the active deposit of thorium emanation. Both source and collector apertures are 6 mm in diameter. The resolution, measured as the ratio of the full width of the curve at half the maximum to the current at the maximum, is found to be $\Delta p/p = 3{\cdot}24\,\%$. For fig. 9 the source is the activated tip of a wire of 0·7 mm diameter. The collector aperture is 1·5 mm in diameter. The resolution is found to be $\Delta p/p = 1{\cdot}47\,\%$. It is apparent from figs. 8 and 9 that source and collector aperture largely determine the resolution. However, the confusion at the image plane, which is due to spherical aberration, is bound to add a further contribution to the width of each transmission curve.

To investigate the aberration of the spectrometer various auxiliary ring and disc baffles were inserted near the centre of the vacuum chamber, leaving in each case an annular slit of 2 mm width only. In a series of measurements the diameter of the annular slit was varied in this way and different small ranges of semi-apertures α were admitted, while the source diameter ($S=0{\cdot}7$mm), the collector aperture diameter ($W=1{\cdot}5$ mm) and the momentum of β-rays (thorium F-line) were unchanged. The transmission curves obtained, corresponding to mean semi-apertures 11° 2′, 9° 22′ and 8° 6′, are plotted in fig. 9. The shift in peak position of these curves is due to spherical aberration. Moreover, if the coil current I_α of each peak is plotted against the square of the semi-aperture α^2 the linear relationship between I_α and α^2 as expected from eqn. (2) can be confirmed and an aberration coefficient $C_s/m \simeq 1{\cdot}5$ can be calculated for the spectrometer according to eqn. (6).

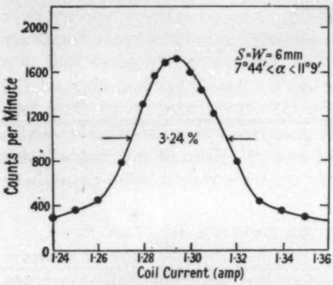

Fig. 8. Transmission curve for thorium F-line. Source and collector aperture both of 6 mm diameter.

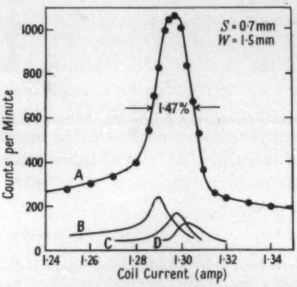

Fig. 9. Transmission curves for thorium F-line. Source diameter $=0{\cdot}7$ mm, collector aperture diameter $=1{\cdot}5$ mm.
Curve A for $7° 44' < \alpha < 11° 9'$.
Curve B for $\alpha = 11° 2' \mp 10'$.
Curve C for $\alpha = 9° 22' \mp 10'$.
Curve D for $\alpha = 8° 6' \mp 10'$.

For the total spread of a transmission curve due to geometry alone, Cosslett (1940 b) has derived recorded limits $\pm\Delta p$ for the thin lens:

$$\Delta p = (S+W)/4D; \qquad \ldots\ldots(18)$$

D is the inner diameter of the annular bundle of rays half way between source and collector aperture. If a solenoid is used it can be shown that $\Delta p/p$ will be twice as large as the value given by eqn. (18). This follows from the proportional relationship (cf. eqn. (13)) which holds between object–image distance in the solenoidal field and the electron momentum ($z_f \propto p$), while eqn. (18) is based on the relation $z_f \propto p^2$, which applies to the thin lens.

The momentum spread for an aberration-free thick lens spectrometer can be obtained from experimental data on the paraxial focusing efficiencies such as are given here by the curves in fig. 5. Differention of eqn. (10) yields

$$\frac{\Delta p}{p} = \frac{\Delta(\text{F.E.})}{(\text{F.E.})}, \qquad \ldots\ldots(19)$$

where
$$\Delta(\text{F.E.}) = \left[\frac{\partial(\text{F.E.})}{\partial(a/z_f)}\right] d\left(\frac{a}{z_f}\right) + \left[\frac{\partial(\text{F.E.})}{\partial(L/z^f)}\right] d\left(\frac{L}{z}\right). \qquad \ldots\ldots(20)$$

In order to evaluate $\Delta(\text{F.E.})$ for the present spectrometer with $a=0{\cdot}155\, z_f$ and $L=0{\cdot}78\, z_f$, the two partial derivatives in eqn. (20) may be obtained from the slopes of the appropriate curves of fig. 3 and fig. 5 respectively; moreover, substitution for

$$d\left(\frac{a}{z_f}\right)=-\left(\frac{a}{z_f}\right)\frac{dz_f}{z_f} \quad \text{and} \quad d\left(\frac{L}{z_f}\right)=-\left(\frac{L}{z_f}\right)\frac{dz_f}{z_f}$$

yields the result

$$\Delta(\text{F.E.})=0{\cdot}21\, dz_f/z_f. \qquad \ldots\ldots(21)$$

Since, however (cf. Cosslett 1940 b),

$$dz_f/z_f=(S+W)/2D, \qquad \ldots\ldots(22)$$

one obtains from eqns. (19), (21) and (22), after substitution of the proper numerical value for F.E., the expression

$$\frac{\Delta p}{p}=\frac{1}{m}\frac{S+W}{2D} \qquad \ldots\ldots(23)$$

with $1/m=0{\cdot}93$.

It may be noted that m in eqn. (23) is identical with the coefficient introduced by eqn. (4) in § 4 since, due to eqn. (5), one has

$$m=\frac{\Delta z_f}{z_f}\frac{I_0}{\Delta I}=\frac{\Delta z_f}{z_f}\frac{(\text{F.E.})}{\Delta(\text{F.E.})}. \qquad \ldots\ldots(24)$$

Now let the diameters of source and collector aperture be approximately equal, and let the mean aperture of the rays admitted by the baffle system be approximately equal to $D=z\,\tan\alpha$. In this case the shape of the resulting transmission curve is such that its half-maximum width is substantially equal to half the total recorded momentum spread ($\pm\Delta p$). Moreover, under these conditions, $\Delta p/p$ as given by eqns. (18) and (23) can be taken to represent the resolution of the aberration-free thin lens and thick lens spectrometer respectively.

Equation (23) is plotted as the straight line A in Fig. 10. In the same graph there are plotted three experimental values for $\Delta p/p$, obtained as half-maximum widths of transmission curves for different sizes of sources and circular collector apertures. These points are seen to lie on a straight line B. Line A passes through the coordinate origin since in an aberration-free spectrometer the width of the transmission curve would vanish ($\Delta p \to 0$) if source and collector aperture were negligibly small. Line B has roughly the same slope as line A, but its abscissae are shifted by 6 mm; hence to $S+W$ there has to be added a further diameter X which must be that of the disc of least confusion due to spherical aberration in the plane of the collector aperture. The value $X=6$ mm extrapolated from curve B is in fair agreement with the diameter of the disc of least confusion given by the following equation of geometrical optics:

$$X=\tfrac{1}{2}\alpha_{\max}\Delta z_f, \qquad \ldots\ldots(25)$$

where $\alpha_{\max}$ is the maximum semi-aperture of the focused rays. Using eqn. (3), eqn. (25) can be written

$$X=\tfrac{1}{2}C_s\alpha^3_{\max}. \qquad \ldots\ldots(26)$$

For the present spectrometer $z_f=1040$ mm and $\alpha_{\max}\simeq 0{\cdot}2$. Moreover, $C_s/z_f m\simeq 1{\cdot}5$ is obtained either from the curves in fig. 2 or from the shift of transmission curves as in fig. 9, and the value $m=1{\cdot}07$ can be determined according to eqn. (24). Substitution of all these values into eqn. (26) yields, as above, the diameter X of the disc of least confusion.

The effect of spherical aberration of the lens upon the resolving power of the spectrometer is largely reduced if, following Frankel (1948), the circular collector aperture is replaced by an annular slot arranged in front of the electron optical image. Hence the use of the ring selector diaphragm mentioned in § 6 leads to a substantial improvement. With the arrangement shown in fig. 7, admitting all apertures of semi-angles between 7·7° and 12·0°, the transmission curve fig. 11 is obtained. There the source has a diameter of 1 mm and the annular selector slit is 1 mm wide, having an outer diameter of 25 mm and an inner diameter of 24 mm. The β-rays are again those of the thorium F line. The resolution obtained from the half-maximum width is $\Delta p/p = 0{\cdot}92\%$. The collecting power calculated from eqn. (17) for the given geometry is $\Omega = 0{\cdot}6\%$ of 4π.

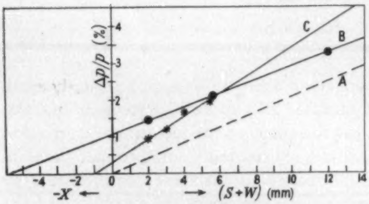

Fig. 10. Resolution, source diameter, collector window and aberration contribution.

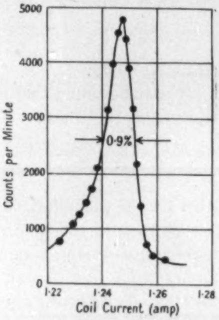

Fig. 11. Transmission curve for thorium F-line taken with ring selector of 1 mm width.

Figure 12 again refers to the study of the effect of spherical aberration when auxiliary disc and ring baffles are inserted near the centre of the vacuum chamber, admitting only rays within narrow angular zones of ±0·4°. The transmission curves for mean semi-aperture angles of 8°, 10° and 11·6° are shown in fig. 12 by the curves A, B and C respectively. The shift of the peaks of these curves demonstrates the spherical aberration. The coil-current at the peak of curve B is seen to be larger than those of curves A and C, demonstrating that when the intermediate bundle (curve B) has its ring focus at the selector slit both the internal bundle (A) and the external bundle (C) pass the plane of the selector slit mainly inside the mean diameter of this slit. If the resolution $\Delta p/p$ of the spectrometer is measured again by the half-maximum width of the transmission curves, a linear relationship is again found to hold near enough between $\Delta p/p$ and $S+W$. This relationship is plotted as the line C in fig. 10, which cuts the abscissa at $X=-1$ mm. A derivation analogous to that yielding eqn. (23) can be applied if W is taken now as the width of the collector slit. X is apparently now the finite width of the ring cross-over of the rays for negligible source diameter. This width is found by extrapolation from line C to be 1 mm, while the diameter of the corresponding focused spot is of the order of 6 mm as shown by the intersection of the abscissa by the line B. The difference in slope of the lines B and

C respectively in fig. 10 is due to a slightly different arrangement of the baffles in these cases.

The two characteristic lines B and C in fig. 10 allow a comparison of the performance of the β-spectrometer with and without the use of the ring selector diaphragm. Under otherwise similar conditions of source and window size ($S \simeq W \simeq 1$ mm) it is seen that the resolution of 1·4% can be improved to 0·8% by the use of the annular selector. This improvement is relatively modest in comparison with gains due to a ring selector reported in the literature for other lens spectrometers, a fact which apparently is due to the limited size of emission angles (semi-apertures 8°–12°) admitted in the present spectrometer. The limitation of the emission angles, however, is inherent in the present design of the high energy spectrometer, where power consumption and weight of the focusing coils are essential considerations.

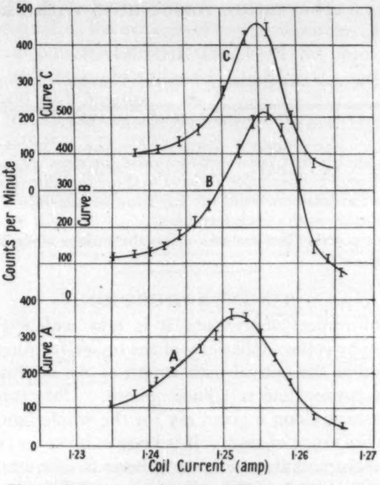

Fig. 12. Transmission curves for three different initial beam apertures.
Ring selector O.D. = 23 mm, I.D. = 21 mm.

Curve A for $\alpha = 8° \mp 0{\cdot}4°$. Curve B for $\alpha = 10° \mp 0{\cdot}4°$. Curve C for $\alpha = 11{\cdot}6° \mp 0{\cdot}4°$.

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The Wave Aberration Associated with Skew Rays

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ABSTRACT. It is shown that invariant foci exist for skew rays, and simple formulae are obtained for the change in aberration relative to those foci due to refraction or reflection at an interface. The formulae are valid for any form of interface. Expressions for focal shifts are obtained together with check formulae.

The method makes possible the analysis of the aberrations along skew rays as they arise at individual surfaces.

§1. INTRODUCTION

In the theory of optical aberrations it is now widely recognized that the significant quantity is the asphericity of the image-forming wave front. This is best measured as the optical path length of the ray intercept between the wave front and an appropriate reference sphere. Unfortunately this quantity is generally not constant along a given ray for the whole family of wave surfaces associated with a given range of rays. It is known, however (Hopkins 1950), that it is constant if the centre of the reference spheres is taken to be at the point of intersection of the given ray and the reference ray. This is a desirable property since the aberration of the given ray is then specified by a unique quantity, independent of the position of the wave front. If now the effect of a refraction, or reflection, is regarded as a transformation of the incident range of rays and the family of associated wave surfaces into the emergent range of rays and its family of associated wave surfaces, the aberration due to the refraction, or reflection, may be defined for each ray to be the increment in the aberration of the wave surfaces consequent upon the transformation. A unique, and meaningful, quantity is thereby obtained which measures the aberration produced on refraction.

In the reference cited above, only the case of intersecting rays was considered, it having appeared not practicable to generalize the definitions and formulae to apply to skew rays. Even so, the restrictive assumption of a spherical refracting, or reflecting, interface was made, and chromatic aberration not treated. It has now been shown that the extension to the case of skew rays is possible, and that general formulae of simple form result.

Any point for which the aberration between two rays of a given pencil remains constant as the wave progresses we shall call an *invariant focus* for the two rays. It will be shown that for any two rays there is an infinity of invariant foci lying in a straight line. If the two rays intersect, the number of such foci is doubly infinite. Moreover the aberration of the family of wave surfaces is found to be independent of whichever of these invariant foci is chosen as the centre of the reference spheres. Accordingly, whenever we speak of the aberration of a ray with respect to a reference ray, we shall imply that the reference sphere is centred on one or other of the invariant foci of the two rays. In other cases we shall speak of the aberration referred to a particular focus.

§ 2. THE INVARIANT FOCI OF TWO SKEW RAYS

In fig. 1, $\bar{A}\,\bar{D}$ and A D are two skew rays whose shortest join is $\bar{D}$ D. F is the mid-point of $\bar{D}$ D. We shall first show that F is an invariant focus for the two rays.

Let $\bar{A}$P, $\bar{B}$Q be two of the wave surfaces associated with the range of rays to which $\bar{A}\bar{D}$, AD belong. $\bar{A}$A, $\bar{B}$B are arcs of two great circles of reference spheres centred on F. To establish the invariant property of this point, we need to prove the equality of the two wave aberrations $\Omega_P = [AP]$, $\Omega_Q = [BQ]$, where square brackets denote optical path lengths.

Since $DF = \bar{D}F$, $BF = \bar{B}F$ and the angles at D, $\bar{D}$ are both right angles, it follows that $BD = \bar{B}\bar{D}$. Similarly $AD = \bar{A}\bar{D}$, and consequently $AB = \bar{A}\bar{B}$. Because $\bar{A}$P, $\bar{B}$Q are two wave surfaces, $PQ = \bar{A}\bar{B}$. Hence $AP = BQ$, and therefore $\Omega_P = \Omega_Q$, the refractive index being the same for both intercepts.

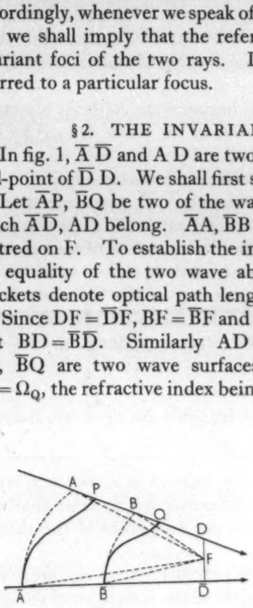

Fig. 1.

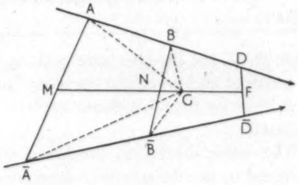

Fig. 2.

Since the mid-point of their shortest join is an invariant focus for the two rays, it might seem appropriate to define this point as their focus. There is an objection to this, however, which is most easily seen by considering the wave front emerging from a telescopic system in which coma is present. A ray emerging from the edge of the aperture in the sagittal section is then skew to the principal ray. The former ray lies in a plane parallel to the meridian plane, and the shortest join between this ray and the principal ray lies in the exit pupil—being in fact, the sagittal radius of this latter. Hence the 'focus' of the two rays would lie in the exit pupil, whereas the region of focus of the whole range of rays is at infinity. This difficulty may be avoided since, as we shall now show, the mid-point of the shortest join of two rays is merely one of an infinity of invariant foci.

In fig. 2, the letters common to this and to fig. 1 signify the same points. M, N are the mid-points of the lines $\bar{A}$A, $\bar{B}$B, respectively. Hence M, N and F are collinear, the line being the skew bisector of the two rays. G is any other point on the skew bisector. We shall now show that G is an invariant focus for the two rays.

From what has been said above, it will be clear that we have only to show that $\mathrm{GA}=\mathrm{G}\overline{\mathrm{A}}$ and $\mathrm{GB}=\mathrm{G}\overline{\mathrm{B}}$. If A, $\overline{\mathrm{A}}$, B, $\overline{\mathrm{B}}$, are each joined to G, the resulting triangles AMG, $\overline{\mathrm{A}}$MG and BNG, $\overline{\mathrm{B}}$NG are congruent, since the skew bisector is perpendicular to each of the joins $\overline{\mathrm{A}}$A, $\overline{\mathrm{B}}$B, and M, N are respectively the mid-points of these latter. Thus $\overline{\mathrm{A}}$, A and $\overline{\mathrm{B}}$, B lie on concentric spheres centred on G. It follows that any point on the skew bisector of two rays is an invariant focus. For a range of rays, the lines of invariant foci of the rays with the principal ray will either intersect or be very close to each other, in the 'waist' of the pencil. A few pairs of rays are therefore needed in order to define a region of focus in the ordinary sense.

If two rays intersect, the line of invariant foci becomes the coplanar bisector of the two rays. However, in this case, all the points of the plane defined by lines equally inclined to the two rays are invariant foci. For skew rays, the number of such foci is infinite; for intersecting rays, the number is doubly infinite.

Since all pairs of reference spheres passing through $\overline{\mathrm{A}}$, $\overline{\mathrm{B}}$, and centred on any given point G of the skew bisector intersect the second ray in the same two points A, B, the wave aberration for this ray is a constant independent of the particular invariant focus to which it is referred. The same result is clearly valid also for the invariant foci of intersecting rays. This property of the invariant foci is important, since, for example, the proofs of aberration formulae may assume the mid-points of the shortest joins as the centres of the reference spheres.

§ 3. ABERRATION FORMULAE

We shall define the aberration produced by an interface between two isotropic media to be

$$\Delta(\Omega)=\Omega'-\Omega \qquad \ldots\ldots(1)$$

where Ω, Ω' are the aberrations along the given ray, relative to a reference ray, of the incident and emergent wave surfaces. The difference $\Delta(\Omega)$, as we shall now show, may be found without explicit reference to the mid-point of the shortest join itself.

The wave aberration produced on refraction or reflection at an interface is expressed by the difference in the optical path lengths of the intercepts of the given ray and the principal ray between the reference sphere for the object space and that for the image space. Since we refer the aberration to invariant foci, any of the object and image reference spheres may be chosen.

Let $\overline{\mathrm{Q}}$Q, $\overline{\mathrm{Q}}'$Q′ (fig. 3) be two reference spheres with centres at F, F′, the mid-points of the shortest joins of the incident and emergent rays respectively. The aberration due to the interface is given by

$$\Delta(\Omega)=[\overline{\mathrm{Q}}\overline{\mathrm{P}}\overline{\mathrm{Q}}']-[\mathrm{QPQ}'] \qquad \ldots\ldots(2)$$

in which Δ denotes the change on refraction, and square brackets are used to denote optical path lengths. $\overline{\mathrm{P}}$, P are the points of incidence of the two rays at an interface separating media of refractive index N, N'. $\overline{\mathrm{Q}}$Q, $\overline{\mathrm{Q}}'$Q′ are great circles of spheres with centres F, F′, and consequently $\mathrm{QD}=\overline{\mathrm{Q}}\overline{\mathrm{D}}$, $\mathrm{Q}'\mathrm{D}'=\overline{\mathrm{Q}}'\overline{\mathrm{D}}'$. The expression (2) is therefore equivalent to

$$\Delta(\Omega)=\Delta(Ne) \qquad \ldots\ldots(3)$$

where $\bar{p}=\overline{\mathrm{P}}\overline{\mathrm{D}}$, $p=\mathrm{PD}$, and $e=\bar{p}-p$.

Let $(\bar{\lambda}, \bar{\mu}, \bar{\nu})$, (λ, μ, ν) be direction cosines of the two incident rays, and let $(\overline{X}, \overline{Y}, \overline{Z})$, (X, Y, Z) be rectangular coordinates of the points of incidence $\overline{\mathrm{P}}$, P.

It is immaterial what coordinate axes are used, and they will therefore not be specified. The lengths $\bar{p}$, p are given by

$$\bar{p}=\Sigma\,\bar{\lambda}(\bar{x}-\bar{X}), \quad p=\Sigma\,\lambda(x-X) \qquad \ldots\ldots(4)$$

$(\bar{x}, \bar{y}, \bar{z})$, (x, y, z) being the coordinates of $\overline{\mathrm{D}}$, D respectively. If U is the angle between the two rays, and $\cos U=\Sigma\,\lambda\bar{\lambda}$, the projection of $\overline{\mathrm{P}}\overline{\mathrm{D}}$ on the ray PD, and of PD on the ray $\overline{\mathrm{P}}\overline{\mathrm{D}}$, gives

$$\bar{p}\cos U=\Sigma\,\lambda(\bar{x}-\bar{X}), \; p\cos U=\Sigma\,\bar{\lambda}(x-X), \qquad \ldots\ldots(5)$$

Using the relationships (4) and (5), together with

$$\Sigma\,\bar{\lambda}(\bar{x}-x)=\Sigma\,\lambda(\bar{x}-x)=0\,,$$

which express the orthogonality of the shortest join and each of the two rays, it is easily shown that

$$e=\frac{\Sigma\,(\lambda+\bar{\lambda})(X-\bar{X})}{1+\Sigma\lambda\bar{\lambda}}, \qquad \ldots\ldots(6)$$

substitution of which in (2) gives

$$\Delta\,(\Omega)=\Delta\left\{N\frac{\Sigma\,(\lambda+\bar{\lambda})\,(X-\bar{X})}{1+\Sigma\lambda\bar{\lambda}}\right\}. \qquad \ldots\ldots(7)$$

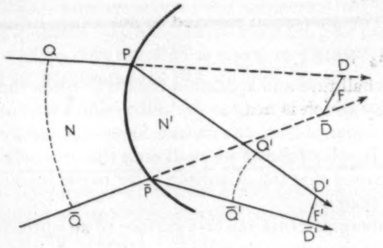

Fig. 3.

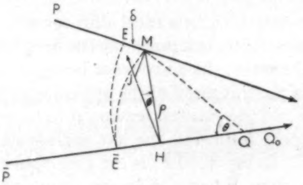

Fig. 4.

It will be noted that (7) involves only the coordinates of the points of incidence of the two rays and their direction cosines, no assumption being made concerning the form of the interface. To apply to cases of reflection, we put $N'=-N$. The data required to calculate (7) are those which are normally available from any ray-tracing computations.* In general a six-figure ray-trace gives adequate accuracy for its evaluation. The aberration of the whole system is obtained by summing the surface contributions. The aberration of the finally emergent wave is thereby obtained, referred to the invariant foci.

In the final image space of an optical system, it is desirable to refer the aberration of all points on the wave surface to a common focus; for example, to the point of intersection of the pupil ray with the gaussian image plane. For this purpose, the position of the wave front must be specified, since the new reference point will not necessarily be an invariant focus. In general, the wave front will be taken to be located in the exit pupil. A focal shift term has then to be added to the aberration. The appropriate formulae are developed in § 4.

The chromatic aberration along a ray may be defined as the optical path length of the intercept between the wave surfaces of two wavelengths λ, $\lambda+\delta\lambda$, assuming the two wave surfaces to be coincident on the reference ray. If $\delta\lambda$ is small, the chromatic aberration $\delta\Omega_\lambda$ is invariant as the wave front progresses. The increment in $\delta\Omega_\lambda$ due to the interface is given by

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* A useful method for skew rays is that given by Smith (1923).

$$\Delta(\delta\Omega_\lambda) = \delta N'(\overline{\mathrm{P}}\overline{\mathrm{Q}}' - \mathrm{PQ}') + \delta N(\overline{\mathrm{Q}}\overline{\mathrm{P}} - \mathrm{QP})$$

or, after a simple reduction,

$$\Delta(\delta\Omega_\lambda) = \Delta(\delta N e) \qquad \ldots\ldots(8)$$

where δN, $\delta N'$ are the increments in refractive index corresponding to a change in wavelength from λ to $\lambda + \delta\lambda$. Substitution of the value of e from (6) in (8) gives

$$\Delta(\delta\Omega_\lambda) = \Delta\left\{\delta N \frac{\Sigma(\lambda + \bar{\lambda})(X - \bar{X})}{1 + \Sigma\lambda\bar{\lambda}}\right\} \qquad \ldots\ldots(9)$$

yielding a simple expression for the chromatic aberration.

§4. FOCAL SHIFT FORMULAE

If the aberration due to each of the surfaces of a system of k surfaces has been found by means of (7), the aberration of the wave surfaces in the final image space is found by the summation

$$\Omega_k' = \sum_{s=1}^{k} \Delta(\Omega_s) \qquad \ldots\ldots(10)$$

and Ω_k' refers to the invariant foci. The aberration referred to any other focus will be given by

$$W_k' = \Omega_k' + \delta W_k' \qquad \ldots\ldots(11)$$

where $\delta W_k'$ is a focal shift term. We shall find an expression for $\delta W_k'$ when the new focus is a point on the reference ray which is near to the diffraction focus of the wave, the wave front being at some distance from the chosen focus—generally in the exit pupil of the optical system. In what follows we shall omit the subscript k and the primes, leaving it to be understood that the quantities refer to the image space following the last surface of the system.

In fig. 4, $\overline{\mathrm{P}}\overline{\mathrm{E}}\mathrm{Q}$ is the principal ray emerging from the last surface of an optical system, $\overline{\mathrm{P}}$ being the point of emergence of the ray at the surface. PEM is another ray of the pencil, P being a point on the last surface. The principal ray cuts the exit pupil of the system at $\overline{\mathrm{E}}$, and $\overline{\mathrm{E}}\mathrm{E}$ is a reference sphere centred on one of the invariant foci of the two rays. The final image surface intersects the principal ray in Q, and $\overline{\mathrm{E}}\mathrm{M}$ is a reference sphere of centre Q. If $\mathrm{EM} = \delta$, positive if M is in advance of E in the direction of propagation of the wave, the focal shift term is given by $\delta W_k' = -N\delta$.

Write $\overline{\mathrm{P}}\overline{\mathrm{E}} = \bar{p}$, $\mathrm{PE} = p$, $\overline{\mathrm{P}}\mathrm{Q} = q$. Then

$$\bar{p} = (\bar{l} - \bar{X})/\bar{\lambda}, \quad p = \bar{p} - e, \quad q = (l - \bar{X})/\bar{\lambda} \qquad \ldots\ldots(12)$$

where $\bar{l}$, l are respectively the X-coordinates of the exit pupil and image, e is the value of e_k' determined by (6) and $(\bar{X}, \bar{Y}, \bar{Z})$ are the coordinates of $\overline{\mathrm{P}}$. The coordinates of M may be expressed in terms of those of $\mathrm{P}(X, Y, Z)$ and $(p + \delta)$. Similarly the coordinates of Q may be obtained from $(\bar{X}, \bar{Y}, \bar{Z})$ and q. Equating $(\mathrm{MQ})^2$ and $(\overline{\mathrm{E}}\mathrm{Q})^2$ then gives

$$\Sigma\{[X + (\bar{p} - e + \delta)\lambda] - [\bar{X} + q\bar{\lambda}]\}^2 = (q - \bar{p})^2$$

of which equation there will be two solutions for δ. We require that for which $\delta \ll (q - \bar{p})$. Hence, omitting δ^2,

$$\delta = \frac{(q - \bar{p})^2 - \Sigma\{(X - \bar{X} - e\lambda) + \bar{p}\lambda - q\bar{\lambda}\}^2}{2\{\Sigma\lambda(X - \bar{X}) + (\bar{p} - e) - q\cos U\}}.$$

This expression simplifies to give

$$\delta W_{k}' = -N\delta = N\frac{\Sigma(X-\overline{X}-e\lambda)^2 - 2(q+\overline{p})\Sigma\overline{\lambda}(X-\overline{X}-e\lambda) + 2q\overline{p}(1-\cos U)}{2\{\Sigma\lambda(X-\overline{X}) + (\overline{p}-e) - q\cos U\}} \qquad \ldots\ldots(13)$$

when the brackets are expanded and the identity $\Sigma\lambda(X-\overline{X}-e\lambda) = -\Sigma\overline{\lambda}(X-\overline{X}-e\lambda)$ is used.

Special cases of (13) arise when either l or $\overline{l}$ is large. Determinate expressions for δ are obtained by dividing the numerator and denominator of (13) by q or $\overline{p}$ in the appropriate cases. It will be noted that the values of q and $\overline{p}$ will never be both infinite, since the final image plane and the exit pupil cannot be simultaneously at infinity.

The focal shift for the chromatic aberration is $-\delta\,.\,\delta N$. This is negligible and we may always write $\delta W_{\lambda}' = \delta\Omega_{\lambda}'$.

If the effects of transverse and longitudinal focal shifts relative to Q, need to be considered, the quantities $1-\cos\theta$ and $\sin\theta\cos\phi$, or alternatively ρ^2 and $\rho\cos\phi$, need to be known for the skew ray. θ is the angular aperture $\mathrm{M\hat{Q}E}$ (fig. 4), ρ the polar distance MH of M from the ray $\overline{\mathrm{P}}\mathrm{Q}$, and ϕ the azimuthal angle of the plane $\mathrm{M\overline{E}Q}$ with respect to the projection plane of the ray $\overline{\mathrm{P}}\mathrm{Q}$ perpendicular to the XY-plane. It will be convenient to give formulae for these quantities here.

The distance PM is equal to $\overline{p}-e+\delta$. Hence, projecting the segments $\overline{\mathrm{P}}\mathrm{P}$, PM, MQ on the ray $\overline{\mathrm{P}}\mathrm{Q}$, we find

$$\Sigma\overline{\lambda}(X-\overline{X}) + (\overline{p}-e+\delta)\cos U + (q-\overline{p})\cos\theta = q\,.$$

It follows at once that

$$1-\cos\theta = \{\Sigma\overline{\lambda}(X-\overline{X}) + (\overline{p}-e+\delta)\cos U - \overline{p}\}/(q-\overline{p})\,. \qquad \ldots\ldots(14)$$

If $\overline{p}$ is large, the numerator and denominator of (14) may be divided by $\overline{p}$ in order to obtain a form more suitable for computation. When $\overline{p}\rightarrow\infty$ the right-hand side of (14) reduces to $1-\cos U$.

We now require an expression for $\sin\theta\cos\phi$. HJ (fig. 4) is perpendicular to the ray $\overline{\mathrm{P}}\mathrm{Q}$, and lies in the projection plane of this latter. J is the foot of the normal from M on this plane and MH is therefore perpendicular to $\overline{\mathrm{P}}\mathrm{Q}$. If (α, β, γ) are the direction cosines of HJ,

$$\alpha = -\frac{\overline{\mu}\overline{\lambda}}{(1-\overline{\mu}^2)^{1/2}},\ \beta = (1-\overline{\mu}^2)^{1/2},\ \gamma = -\frac{\overline{\mu}\overline{\nu}}{(1-\overline{\mu}^2)^{1/2}} \qquad \ldots\ldots(15)$$

the positive square root being taken, since we require HJ to have the same sign as its projection on the Y-axis. It may be noted that $\alpha = -\overline{\mu}$, $\beta = \overline{\lambda}$, $\gamma = 0$ when the ray $\overline{\mathrm{P}}\mathrm{Q}$ lies in the XY-plane. Moreover, if this ray makes an angle $\overline{U}$ with the X-axis, $\overline{\lambda} = \cos\overline{U}$, $\overline{\mu} = -\sin\overline{U}$, $\overline{\nu} = 0$, the angle $\overline{U}$ having the sign customarily used in optics.

Writing $f = \overline{\mathrm{P}}\mathrm{H}$, the coordinates of H are equal to $\overline{X}+f\overline{\lambda}$, $\overline{Y}+f\overline{\mu}$, $\overline{Z}+f\overline{\nu}$. Hence, projecting the segment HM on the direction (α, β, γ),

$$\begin{aligned}\mathrm{HJ} &= \Sigma\alpha\{[X+(\overline{p}-e+\delta)\lambda] - [\overline{X}+f\overline{\lambda}]\}\\ &= \Sigma\alpha(X-\overline{X}) + (\overline{p}-e+\delta)\Sigma\lambda\alpha\,,\end{aligned}$$

one group of terms vanishing because $\Sigma\overline{\lambda}\alpha = 0$. Since $\mathrm{HJ} = (\mathrm{MQ})\sin\theta\cos\phi$, we find

$$\sin\theta\cos\phi = \{\Sigma\alpha(X-\overline{X}) + (\overline{p}-e+\delta)\Sigma\lambda\alpha\}/q-\overline{p})\,. \qquad \ldots\ldots(16)$$

If $\overline{p}\rightarrow\infty$, the right-hand side of (16) reduces to $-\Sigma\lambda\alpha$.

The expressions (14) and (16) are those most commonly used for calculating the effects of focal shifts. If, however, the image is at a great distance from the optical system, q is large and, in place of $1-\cos\theta$ and $\sin\theta\cos\phi$, the values of ρ^2 and $\rho\cos\phi$ are required. These may be found as follows.

The angle $\overline{\mathrm{E}}\widehat{\mathrm{M}}\mathrm{H}$ (fig. 4) is equal to $\theta/2$, and therefore

$$\rho^2 = \tfrac{1}{2}(1+\cos\theta)\,G^2, \qquad \ldots\ldots(17)$$

where $G = \overline{\mathrm{E}}\mathrm{M}$. The lengths $\overline{\mathrm{PE}}$, PM are respectively equal to $\bar{p}$ and $\bar{p}-e+\delta$. Hence

$$G^2 = \Sigma\{(X-\overline{X}) + \bar{p}(\lambda-\bar{\lambda}) - (e-\delta)\lambda\}^2. \qquad \ldots\ldots(18)$$

The value of $\cos\theta$ is obtained from (14), and this together with (17) and (18) gives the value of ρ^2. The quantity $\rho\cos\phi$ may be seen from (16) to be given by

$$\rho\cos\phi = \Sigma\alpha(X-\overline{X}) + (\bar{p}-e+\delta)\,\Sigma\lambda\alpha \qquad \ldots\ldots(19)$$

since $\rho = (q-\bar{p})\sin\theta$.

In the general case ρ and $\sin\theta$ will be positive, since they are polar distances of M with respect to the ray $\overline{\mathrm{P}}\mathrm{Q}$. In the projection plane of $\overline{\mathrm{P}}\mathrm{Q}$ $(\phi=0,\ \pi)$, we may conveniently omit explicit reference to the azimuth ϕ, and allow ρ, $\sin\theta$ to have positive or negative values according as M is above or below the ray $\overline{\mathrm{P}}\mathrm{Q}$. The signs given by (16) and (19) will then agree with this convention.

§ 5. CHECK FORMULAE

In applications of the above formulae to the designing of optical systems, a substantial amount of further computation may be undertaken as a result of the indications of the analysis of the aberrations. In consequence, it is desirable to have check formulae available.

An approximate check for the values of e, e' at different surfaces is obtained by noting that $D_p = \overline{D}_p + e_{p+1} - e_p'$ for the space between surfaces p and $p+1$. D, $\overline{D}$ are the intercepts of the two rays between these surfaces. In general, the values of e, e' will be known to at least one place greater accuracy than D, $\overline{D}$, and consequently the check is not fully accurate. However, a six-figure ray-tracing calculation will generally give values of D, $\overline{D}$ which are in error by no more than about $\lambda/10$, λ being the wavelength of D-light. Indeed, in preliminary work, one may use the identity

$$\Sigma\Delta(Ne) = N_1e_1 + \sum_{1}^{k-1} N_p(\overline{D}_p - D_p) + N_k'e_k' = \Omega_k' \qquad \ldots\ldots(20)$$

to calculate the final aberration of the system. This corresponds with the well-known formula

$$\delta W_\lambda' = \delta N_1e_1 + \sum_{1}^{k-1} \delta N_p(\overline{D}_p - D_p) + \delta N_k'e_k'$$

for the chromatic aberration, although this latter is of greater accuracy on account of the smallness of δN. It does not seem easy to construct checks for the values of e_1, e_k'. Since only two quantities are involved, however, it is reasonable to check them by a recomputation, following a different procedure.

To check the values of $(1-\cos\theta)$ and $\sin\theta\cos\phi$, or alternatively of ρ^2 and $\rho\cos\phi$, we may obtain an expression for the length JM (fig. 4). If (i, j, k) are the direction cosines of JM,

$$i = -\bar{\nu}/(1-\bar{\mu}^2)^{1/2}, \quad j=0, \quad k=\bar{\lambda}/(1-\bar{\mu}^2)^{1/2} \qquad \ldots\ldots(21)$$

since JM is perpendicular to the projection plane of $\overline{\mathrm{P}}\mathrm{Q}$. If this ray lies in the XY-plane, $\bar{\nu}=0$ and $i=j=0$, $k=1$. In (21) the positive value of the square root is taken. Projecting the length HM on the direction (i, j, k) and writing $\overline{\mathrm{P}}\mathrm{H} = f$,

$$\begin{aligned} \mathrm{JM} &= \Sigma i\,\{[X + (\bar{p} - e + \delta)\lambda] - [\bar{X} + f\bar{\lambda}]\}. \\ &= \Sigma i\,(X - \bar{X}) + (\bar{p} - e + \delta)\,\Sigma\lambda i, \end{aligned}$$

one group of these terms vanishing because $\Sigma\bar{\lambda}i = 0$. Since $\mathrm{JM} = (\mathrm{MQ})\sin\theta\sin\phi$ we find

$$\sin\theta\sin\phi = \{\Sigma i\,(X - \bar{X}) + (\bar{p} - e + \delta)\,\Sigma\lambda i\}/(q - \bar{p}). \qquad \ldots\ldots(22)$$

The sum of the squares of (16) and (21) now gives a value for $\sin^2\theta$, which may be used to check the value of $1 - \cos\theta$ given by (14).

If the alternative formulae (17) and (19) are used, (22) may be written

$$\rho\sin\phi = \Sigma i\,(X - \bar{X}) + (\bar{p} - e + \delta)\,\Sigma\lambda i \qquad \ldots\ldots(23)$$

The sum of the squares of (19) and (23) now gives a value of ρ^2, which may be compared directly with that given by (17).

By means of the value of $1 - \cos\theta$, or alternatively ρ^2, it is possible to check the value of the focal shift $\delta W_k'$ given by (13). In fig. 4, Q_0 is a point on the ray $\overline{\mathrm{P}}\mathrm{Q}$ such that $\mathrm{EQ}_0 = \overline{\mathrm{E}}\mathrm{Q}_0$. Hence the aberration of the wave front which passes through $\overline{\mathrm{E}}$ will have the same value with reference to the focus Q_0 as when measured with reference to the invariant foci—although of course Q_0 will not be an invariant focus for the two rays unless the two rays intersect in this point. The difference between the aberrations Ω_k', W_k' is thus equal to the focal shift from Q_0 to Q. This may be easily found if the distance $\overline{\mathrm{P}}\mathrm{Q}_0 = q_0$ is known.

The coordinates of E and Q_0 are respectively $\{X + \lambda p,\ Y + \mu p,\ Z + \nu p\}$, $\{\bar{X} + \bar{\lambda}q_0,\ \bar{Y} + \bar{\mu}q_0,\ \bar{Z} + \bar{\nu}q_0\}$. Hence, equating the squares of EQ_0 and $\overline{\mathrm{E}}\mathrm{Q}_0$,

$$\Sigma\{(\bar{X} + \bar{\lambda}q_0) - (X + \lambda p)\}^2 = (q_0 - \bar{p})^2.$$

The terms in $q_0{}^2$ cancel, leaving after some reduction

$$q_0 = \frac{(\bar{p}^2 - p^2) + 2p\Sigma\lambda(\bar{X} - X) - \Sigma(\bar{X} - X)^2}{2\{\bar{p} - p\cos U + \Sigma\bar{\lambda}(\bar{X} - X)\}}$$

Replacing p by $\bar{p} - e$, we find

$$q_0 = \frac{(p - e)\{e - \Sigma\lambda(X - \bar{X})\} + \frac{1}{2}e^2 - \frac{1}{2}\Sigma(X - \bar{X})^2}{\bar{p}(1 - \cos U) + e\cos U - \Sigma\bar{\lambda}(X - \bar{X})} \qquad \ldots\ldots(24)$$

in which, as before, $\cos U = \Sigma\lambda\bar{\lambda}$ is the cosine of the angle between the two rays. If the exit pupil is at a great distance from the system, the numerator and the denominator of (24) may be divided by $\bar{p}$ in order to obtain a form more suitable for computation.

The change in focus from Q_0 to Q will change the radius of the reference sphere from $(q_0 - \bar{p})$ to $(q - \bar{p})$. In consequence the change in aberration is

$$\delta W_k' = -N(1 - \cos\theta)\,\frac{q - \bar{p}}{q_0 - \bar{p}}\,(q_0 - q) \qquad \ldots\ldots(25)$$

which may be compared with the value given by (13). If q is large, the expression (24) may be inverted and the reciprocal $1/q_0 = g_0$ calculated. The focal shift is then given by

$$\delta W_k' = -\tfrac{1}{2}NG^2\left(\frac{1}{q - \bar{p}} - \frac{1}{q_0 - \bar{p}}\right),$$

or

$$\delta W_k' = -\tfrac{1}{2}NG^2\left(\frac{g}{1 - g\bar{p}} - \frac{g_0}{1 - g_0\bar{p}}\right) \qquad \ldots\ldots(26)$$

where $g = 1/q$.

The expression for the focal shifts given here differ slightly from those given previously (Hopkins 1950). The earlier formulae assumed $\delta R \ll R$, R being the radius of the reference sphere, and δR the change in radius. The present formulae are more precise if $\delta R/R$ is not very much less than unity. The case may arise if, for example, the pencil is of very small angular aperture, although the difference between the two sets of formulae is rarely very significant.

§ 6. APPLICATIONS

The ray intersection points with the final stage surface measure the derivative of the wave aberration, but not this quantity itself. By means of the present formulae both the aberration and its derivative are obtained from one ray trace. In consequence, the state of correction of an optical system may be more easily ascertained. Moreover, the aberration along a given ray is expressed as the sum of surface contributions, and this facilitates the reduction, or balancing, of higher order aberrations by indicating the surfaces which make larger contributions to the final aberration, the conditions of refraction at the surface needing then to be modified. This has not previously been possible for skew rays, or for refracting surfaces other than spheres. In addition, chromatic aberration may also be analysed into surface contributions, a useful technique which was not possible with the earlier results.

These latter formulae (Hopkins 1950)—due originally to Conrady (1923)—give, for spherical surfaces and co-planar rays, formulae whose accuracy is a good deal greater than the ray-tracing data which they involve. Against this may be cited their complexity, involving trigonometrical functions of sub-multiples of the differences and sums of angles, even in the simple case for which they are known. For skew rays, or non-spherical surfaces, greatly increased complexity would result and it seems that the formulae, if obtained, would be of little use on this account. The present formulae give adequate accuracy in most cases from a six-figure ray-trace. In other cases, an additional place in the ray-tracing involves far less effort than the computation of the older formulae, even in the cases where they are known.

The formulae given in the present communication may be applied to the study of the aberrations of any optical system. For example, the correction of non-symmetrical systems involving prisms, refracting or reflecting cylindrical or toric surfaces, may easily be treated. Of particular importance would seem to be the use of the method for the investigation of the effects of non-spherical surfaces in symmetrical optical systems, and their use in conjunction with differential correction formulae for wave aberrations.

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Ultrasonic Absorption in Mixtures of Ethyl Alcohol and Water

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ABSTRACT. The pulse method has been used to measure the absorption of ultrasonic waves in mixtures of ethyl alcohol and water at the three frequencies 22·5, 37·5 and 52·5 Mc/s and at the temperatures 0°, 10° and 25° c. The results confirm the findings of earlier workers that in both the pure liquid and the mixture there is more absorption than can be explained by the classical mechanisms of viscosity and heat conduction. When the composition of the mixture is varied a maximum of absorption is found for a mixture containing 20 to 30 molar per cent alcohol. The magnitude of the absorption at the maximum increases rapidly as the temperature is lowered. Over a range of compositions in the neighbourhood of the maximum the value of α/f^2 (α is the absorption coefficient and f the frequency) is found to decrease as f increases.

It is known that the extra absorption in the pure liquids can be explained in terms of a compressibility with a finite relaxation time, but this mechanism alone cannot account for the behaviour of the mixture. It is suggested that here the additional absorption is due to interaction between the molecules of the two components, leading to the partial development of a structure or complex in the liquid with a composition of four molecules of water to one of alcohol. The thermal properties of the mixture indicate that such interaction occurs, and the suggestion for the composition of the complex is supported by the fact that the freezing-point curve for the mixture shows that a structure with this composition is formed in the solid phase.

The extra absorption found by other workers in several similar mixtures can be explained, at least qualitatively, in the same way.

§ 1. INTRODUCTION

When the absorption of ultrasonic waves is measured in mixtures of some liquids it is found that the absorption coefficient reaches a maximum value for some particular constitution of the mixture. Thus in a mixture of ethyl alcohol and water the maximum is reached when the mixture contains about 23% alcohol. No explanation of this phenomenon has yet been published, but the author has had the privilege of discussing it theoretically with E. Bauer, and from the discussions it became apparent that more detailed experimental results were required before theories could be properly tested. It is the purpose of this paper to present these results and to discuss their significance.

The previous measurements on mixtures of polar liquids (Willard 1941, Willis 1947, Burton 1948, Sette 1951) had shown that only some combinations exhibit the maximum which it is the purpose of this paper to study. Those mixtures which did show a maximum had been investigated only at room temperature, and by techniques insufficiently accurate to determine how the absorption varied with frequency. In the present work measurements have been made at two temperatures (with a few measurements at a third temperature) and on three frequencies in an attempt to outline the way in which the phenomenon changes with temperature and frequency. A mixture of ethyl alcohol and water was used for the investigation.

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§ 2. METHODS AND RESULTS

The absorption measurements were made by the pulse method (Pellam and Galt 1946), which is thought to be the most accurate of the methods at present available. The experimental equipment was taken over complete from Dr. J. M. M. Pinkerton, who has already published a full account of it (Pinkerton 1949 b). There is therefore no need to describe the method here, but it should be stated that the measurements were made on a pulse which had travelled about 1 cm in the liquid. A thermostat kept the temperature of the liquid uniform to about 0·1° c. Chemically pure absolute alcohol and distilled water were used in preparing the mixtures.

The absorption coefficient was measured for mixtures with compositions ranging from pure water to 85% alcohol. The measurements were made at the three frequencies 22·5, 37·5, and 52·5 Mc/s for the two temperatures 0° c and 25° c. In addition a limited number of measurements were made at 10° c.

Some of the results are shown in the diagrams. The quantity plotted vertically is α/f^2, where α is the pressure absorption coefficient and f the frequency, since for pure viscous absorption this quantity would be independent of frequency. The accuracy of the measurements, estimated from their self-consistency, was within about 1%.

The full curve in fig. 1 shows how, for a fixed temperature and frequency, the absorption coefficient varied with the composition of the liquid. In fig. 2 three of these curves, plotted for different frequencies but the same temperature, are compared, and in fig. 3 curves obtained at the same frequency but at different temperatures.

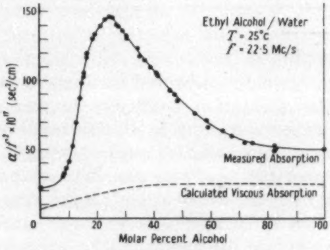

Fig. 1.

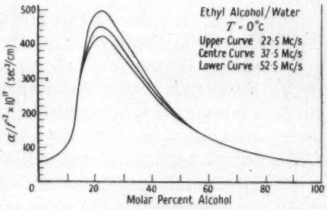

Fig. 2.

§ 3. DISCUSSION OF RESULTS

The classical (Stokes) theory of the absorption of sound in liquids ascribed it to the action of shear viscosity. This theory showed that the quantity α/f^2 would be expected to be constant. It has however long been known that only in a few liquids can the whole of the absorption be explained in this way, and that in most the absorption is much larger than the classical value.

Recent work has shown that such excess absorption can occur as a result of several different processes which might have relaxation times comparable with the period of the sound wave. These all involve the existence in the liquid of some kind of statistical equilibrium between states of different energy which is perturbed by the variations of temperature or pressure accompanying the wave.

The behaviour of a liquid containing an equilibrium sensitive to the temperature variations can be described by means of a specific heat with a relaxation time, and similarly a liquid with an equilibrium perturbed by the variations of pressure displays a compressibility with a relaxation time. Systems in which there is a statistical equilibrium between either the vibrational and other degrees of freedom of the molecule or chemical compounds are associated with a specific heat which shows a relaxation time. A compressibility with a relaxation time is found in systems in which an equilibrium exists between structural states of different density. It is characteristic of the absorption produced by structural relaxation that its ratio to the classical absorption is independent of temperature, and both decrease as the temperature increases. On the other hand absorption due to thermal relaxation commonly increases slightly with increasing temperature. In all these types of system the absorption varies with frequency according to the relaxation law (Kneser 1938) $\alpha/f^2 = A/[1+(f/f_m)^2]$, where f_m is the characteristic relaxation frequency for the process. In most liquids, however, the relaxation frequencies are well above the range of measurement of existing apparatus, so that through this range α/f^2 is constant as for pure viscous absorption.

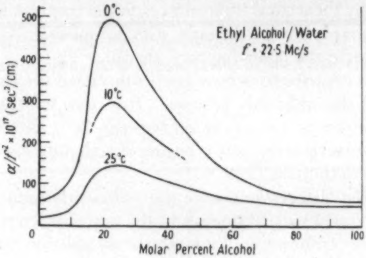

Fig. 3.

The particular type of equilibrium which may exist in a given liquid depends on its chemical composition (Pinkerton 1949 a). Absorption due to the slowness of interchange of energy between the vibrational and other degrees of freedom is found only in non-polar liquids and their mixtures (Bauer 1949). In polar liquids the interaction between molecules is too strong, and collisional de-excitation therefore too rapid for relaxation of their internal degrees of freedom to play any part in producing absorption. This same interaction may, however, result in the formation of a relatively open pseudo-crystalline structure in the liquid which, through its equilibrium with a denser, more closely packed arrangement of molecules, gives rise to a compressibility with a relaxation time.

Hall (1948) has shown that in pure water and pure alcohol the excess absorption over and above that due to viscosity can be explained by this mechanism of structural relaxation. In mixtures of the two, however, the total absorption is greater than the proportioned sum of the absorptions of the constituents, and has a maximum at a composition between 20% and 30% alcohol. At and around the maximum the quantity α/f^2 depends on the frequency, whereas for pure water and pure alcohol it is independent of frequency. The magnitude of the absorption at the maximum and variation of α/f^2 with frequency both increase as the temperature is reduced. At intermediate compositions the ratio of the observed to the classical absorption depends on the temperature, whereas in the pure liquids it is independent of temperature, as expected if the extra absorption were due to a compressibility with a relaxation time.

Clearly some process besides the two already mentioned is acting to produce an extra absorption in the mixture, and the first step in studying this process must be to decide how much it contributes to the absorption.

The absorption due to shear viscosity is given by the expression

$$\alpha/f^2 = 8\pi^2\eta/3\rho v^3,$$

and in computing this contribution the figures for the viscosity η and the density ρ were taken from the *International Critical Tables* and those for the velocity v from some measurements of Giacomini (1949). In some cases it was necessary to extrapolate for the figures at 0° c from values at 5° or 10° c.

A typical example of the way in which the calculated viscous absorption depends on composition is shown in the lower (broken) curve in fig. 1.

It is much more difficult to decide what is the contribution of that part of the absorption produced by a compressibility with a relaxation time (Hall absorption). In the absence of any theoretical indication as to the magnitude of this absorption it has been taken simply to vary linearly with composition from the one known value for pure water to the other for pure alcohol.

When these two contributions have been subtracted the remainder represents the contribution of the unknown process. It is zero for the two pure liquids, and reaches a maximum at a composition between 20% and 30% alcohol. The maximum in the measurements at 0° c occurs at a slightly lower concentration of alcohol than it does in those at 25° c.

Unfortunately the observations were not sufficiently accurate or extensive to show whether α/f^2 varied with frequency in the way characteristic of one or more relaxation processes. If, however, we make the assumption that the absorption is all due to a single relaxation mechanism, then its relaxation frequency can be calculated from the observed way in which α/f^2 varies with frequency. The equation which expresses the variation of the absorption with frequency may be written in the form

$$f^2/\alpha = A^{-1}[1+(f/f_m)^2],$$

so that a graph in which f^2/α is plotted as a function of f^2 should be a straight line, and the value of f_m can be found from its intercept on the horizontal axis. The values of the relaxation frequency obtained in this way for the maxima of the absorption curves are 103 ± 5 Mc/s for 0° c, 22% alcohol, and 145 ± 10 Mc/s for 25° c, 25% alcohol. The principal source of error in these figures is the uncertainty in the allowance for the Hall absorption, and the limits quoted allow for a possible 50% error in this quantity.

The accuracy of the measurements is not sufficient to show how the relaxation frequency depends on composition. In the measurements at 0° c it appears however to increase rapidly on going away from the maximum in the direction of increasing alcohol content.

§ 4. THE CAUSE OF THE EXTRA ABSORPTION

We may now proceed to the problem of inferring the nature of the process responsible for this excess absorption. It has only so far been found in mixtures of polar liquids (Willard 1941, Willis 1947, Burton 1948, Bauer 1949, Sette 1951).

This limitation, together with the fact that the absorption reaches a maximum at an intermediate composition, suggests that the process may be related to an interaction between the molecules of the two species which results in the formation of a compound structure or complex in the liquid, in such a way as to give a contribution to the specific heat with a finite relaxation time. The complex must be thought of as a state of local short-range order rather than as a definite chemical compound, and it is the extent of this local order which would be perturbed by the temperature variation in the sound wave, so as to produce absorption.

This possibility was considered by Burton (1948), but he rejected it because neither an x-ray diffraction experiment nor dielectric polarization measurements indicated any degree of molecular interaction in the mixture. It would, however, be most surprising if there were no interaction, in view of the fact that both of the pure liquids are associated and that their molecules are polar. Possibly the measurements concerned were taken at too high a temperature to reveal the association of the mixture.

More recently the matter has been considered in some unpublished work by Bauer, which the author has had the advantage of seeing. Bauer has suggested that evidence for association may be found in the two facts (*a*) that the heat of mixing is finite and (*b*) that the specific heat of the mixture does not depend linearly on composition. The way in which the heat of mixing varies with composition is shown in fig. 4, curve A, and it can be seen that, like the absorption, it has a maximum at a composition of about $20\%_c$ alcohol. The thermal data required for the construction of this and subsequent figures were mostly obtained from the *Handbook of Chemistry and Physics* (Chemical Rubber Publishing Co.) and the *International Critical Tables*.

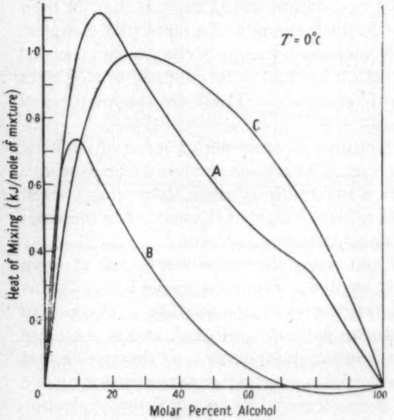

Fig. 4. Curve A ethyl alcohol/water; curve B *n*-propyl alcohol/water; curve C methyl alcohol/water.

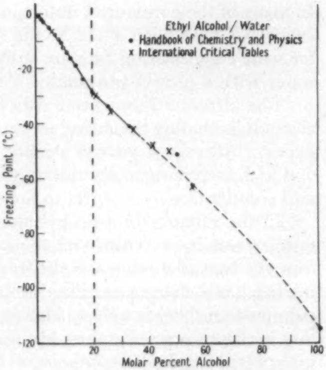

Fig. 5.

Qualitatively, the formation of a complex explains most of the observations. It explains the way in which the absorption decreases with increasing temperature, since the structure of the complex will be broken up by thermal agitation so that at high temperatures there is less of it present in the mixture. It explains also the presence of the maximum in the absorption curve, which should occur roughly at the composition where the proportions of the constituents in the mixture are equal to those in the complex. In the measurements at 25° c the maximum occurs at about 25% alcohol, but in those at 0° c, where the structure is supposed to be better developed, it occurs at about 22%. This suggests that the true composition of the complex is 20% alcohol, or four molecules of water round each molecule of alcohol.

Supporting evidence for the existence of this complex is provided by the freezing-point curve for the mixture. We have already seen that the proportion of the complex present would be expected to increase as the temperature was decreased, so that it seems reasonable to suppose that the structure would be fully developed in the solid state. If this were so the freezing-point curve would have a discontinuity in its slope at the composition corresponding to that of the complex. The available freezing-point data for a mixture of ethyl alcohol and water are plotted in fig. 5, and in fact a kink is found in the curve at the composition corresponding to 20% alcohol.

§ 5. ABSORPTION IN OTHER MIXTURES

It is interesting to see whether a similar correlation between the ultrasonic absorption and the thermal properties can be found in any of the other mixtures which have previously been studied. A correlation would suggest that, in their cases also, the extra absorption was due to the formation of a molecular complex. In many of these mixtures, unfortunately, no measurements of the relevant thermal properties have been made, so the discussion has had to be confined to two cases for which the existing data are sufficiently complete. These are the mixtures of water with *n*-propyl and methyl alcohols.

The ultrasonic absorption data for mixtures of water with a series of aliphatic alcohols, including the above, are shown in fig. 6, which is reproduced from Burton's paper. All except methyl alcohol have a maximum in their absorption curves, and with increasing molecular weight the relative height of the maximum increases and simultaneously it shifts to lower concentrations of alcohol.

In the mixture of *n*-propyl alcohol and water the maximum occurs at room temperature in a mixture with about 14% alcohol. Figures 4, curve B, and 7 show how the heat of mixing and the freezing point vary in this mixture. The heat of mixing has a sharp maximum for a mixture with 10% alcohol, and at the same composition there is a sharp kink in the freezing-point curve. It therefore seems that in this mixture also the additional absorption may be due to the formation of a molecular complex, its composition being nine molecules of water to one of alcohol.

The mixture of water and methyl aclohol, as already mentioned, has no maximum in its absorption curve at room temperature. Correspondingly, we find that although the heat of mixing is finite (fig. 4, curve C), implying that interaction between the components of the mixture does take place, there is no sharp peak in its variation with composition. Likewise the freezing-point curve (fig. 8) shows that in the solid no compound structure is formed in the range 0–60% alcohol. At 60%, however, the freezing point of the mixture is lower than that of the pure alcohol, and so the probability is that a complex is formed at a higher alcohol concentration. Unfortunately the measurements do not cover this range.

We have thus to ask why this complex does not produce a maximum in the absorption curve. We first notice that the freezing-point curve indicates that the melting point of the complex is below $-100°$ C, whereas the ethyl alcohol–water complex melts at $-29°$ C and the *n*-propyl alcohol–water complex at $-10{\cdot}3°$ C. It may well be therefore that the complex formed between methyl alcohol and water is completely destroyed by thermal agitation at room temperature. If this were so one would expect that at lower temperatures the absorption curve for the mixture of methyl alcohol and water would develop a maximum like the rest.

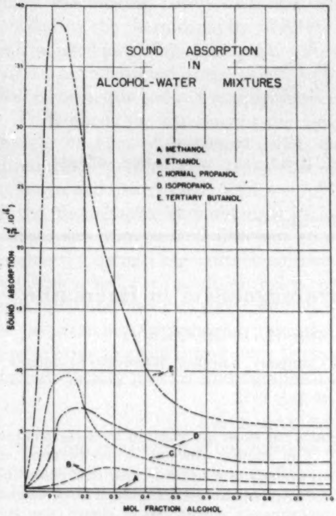

Fig. 6.

With acknowledgment to the 'Journal of the Acoustical Society of America'.

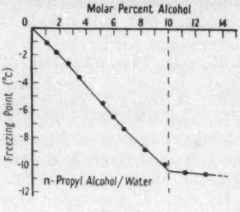

Fig. 7.

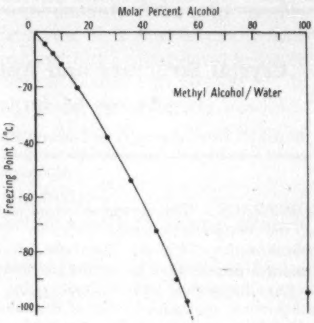

Fig. 8.

Since the behaviour of the other alcohols studied by Burton is similar it seems reasonable to generalize and to suggest that in the whole of this series of mixtures the extra absorption is due to the persistence into the liquid phase of structures or complexes which are fully developed in the solid state. The coordination number of the alcohol molecule in the complex increases with increasing molecular weight.

As yet this suggestion is purely qualitative. There would be value in a quantitative theory of the formation of such complexes in the liquid state, which no doubt would have to be treated as a cooperative phenomenon. On the practical side it would be of interest to have measurements of the freezing points of some of the other mixtures whose absorptions have already been determined.

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Crystal Structure and Antiferromagnetism in Haematite

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ABSTRACT. The crystal structure of haematite has been investigated by x-ray analysis in the temperature range 20° c to 950° c. On cooling through a temperature of approximately 675° c the rhombohedral unit cell undergoes a sudden expansion along the triad axis direction. The results are interpreted in terms of an antiferromagnetic behaviour of pure haematite, with a Curie point at approximately 675° c. It is shown that the behaviour is analogous to that of the cubic oxides FeO and Fe_3O_4, which exhibit structure cell deformations on cooling through certain critical temperatures.

§ 1. INTRODUCTION

The magnetic properties of haematite, α-Fe_2O_3, undergo certain changes in the neighbourhood of 675° c, which have been discussed in detail by Chevallier (1951). At this same temperature a thermal expansion abnormality has been reported by Chaudron and Forestier (1924), and Roth and Bertram (1929) have observed an anomaly in the specific heat–temperature curve. Thus a second-order change takes place at 675° c, and Néel (1949) has suggested that this change comprises a transition from an antiferromagnetic to a paramagnetic state. Recent neutron diffraction work on haematite by Shull *et al.* (1951) has given some support to Néel's hypothesis.

In view of the absence of information concerning the precise behaviour of the structure cell of haematite near the critical point of 675° c, it was decided to investigate the change of the lattice parameters of the material over the temperature range 20° c to 950° c. The results prove that the structure cell of α-Fe_2O_3

undergoes a deformation resembling those exhibited by FeO and Fe_3O_4 at certain critical temperatures, and it is concluded that this is consistent with the view that haematite assumes an antiferromagnetic state below 675° c.

§ 2. APPARATUS AND SPECIMENS

The x-ray camera used for the investigation was a high temperature 19 cm Debye–Scherrer camera of the design previously described by Owen (1943). The temperature of the sample could be varied from room temperature to 950° c, and maintained constant to $\pm 1^\circ$ c. An earlier calibration of the instrument against the melting points of several metals had shown that the temperatures recorded by the thermocouples were within a few degrees of the temperature of the irradiated part of the specimen. Further accuracy in measuring the temperature was not attempted, since, as will become apparent in the discussion, a possible small error in our scale of temperatures has no important bearing on the results.

Throughout the experiment the air inside the camera was maintained at a pressure of 1 cm of mercury. This pressure was sufficiently low to obtain reproducible performance from the furnaces within the high temperature enclosure, and also to reduce background fogging on the x-ray powder photographs. All the photographs were obtained with Co $K\alpha$ radiation.

Samples from several different sources were studied. The chief samples comprised a natural haematite from Sweden, a natural micaceous haematite from East Africa, and a red synthetic oxide, prepared by firing a precipitated hydrated oxide (goethite) in air at 400° c and then for several hours in air at 1300° c. In the preparation of specimens from the hard natural materials care was taken in powdering to avoid introducing lattice strains. All the specimens investigated gave satisfactory powder photographs exhibiting no appreciable line broadening.

§ 3. X-RAY EXAMINATION

At room temperatures haematite is rhombohedral, with the oxygen atoms in approximately hexagonal close packing and the iron atoms occupying two-thirds of the octahedral holes in the oxygen lattice (Wells 1950). It will be convenient to describe the results with reference to a hexagonal, rather than a rhombohedral, structure cell.

A preliminary exploration of the thermal changes in the lattice was made without recourse to detailed measurements at every temperature. Powder lines were sought whose relative positions were particularly sensitive to any change of shape of the structure cell, i.e. change of c/a. It so happens that the adjacent high-order lines 10(14) and 410 are very favourably placed for this purpose.

The relative positions of these two lines at different temperatures in the range 20° c to 950° c are shown in the photographs (actual size) in fig. 1. It can be seen that the separation of the lines Δ changes in a non-uniform manner with temperature. From 20° c to 300° c the change of Δ is relatively small, but from 300° c to 680° c Δ decreases more rapidly, and at a rate which increases with temperature. Above 680° c Δ is again almost independent of temperature.

The general shape of the (Δ, temperature) curve was closely similar to that of the (c/a, temperature) curve, and the latter is shown in fig. 2. This curve possesses a point of inflection at approximately 675° c, which is the critical temperature for the second-order change. Within the accuracy of measurement of c/a ($\pm 0{\cdot}0002$) it was found that the value of c/a for a given sample in the temperature range 20° c to 950° c was independent of the previous heat treatment in this range.

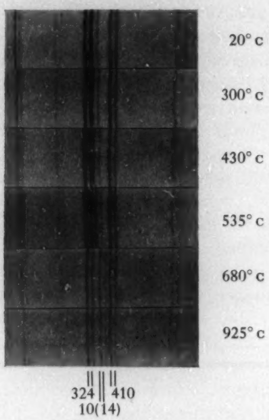

Fig. 1. Parts of x-ray powder photographs of haematite at successively increasing temperatures showing relative displacements of 324, 10(14) and 410 reflections.

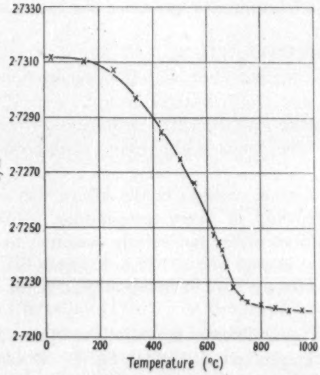

Fig. 2. Curve showing change with temperature of axial ratio c/a of structure cell of haematite.

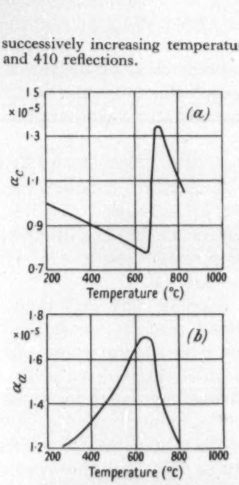

Fig. 3. Curves showing variations with temperature of expansion coefficients of haematite along (*a*) *c* direction, (*b*) *a* direction.

Several other synthetic oxides were examined, and all gave substantially the same curve as fig. 2. Some slight differences in the room temperature value of c/a were observed for certain specimens, and in particular for synthetic oxides fired at relatively low temperatures the measured value was $2{\cdot}7305$ instead of

2·7311. Subsequent heat treatment of such specimens in air at 1300° C, however, led to an increase of the room temperature value of c/a to 2·7311. Thus fig. 2 represents the general character of the (c/a, temperature) curve for all the samples of haematite examined.

The expansion coefficient–temperature curves for the directions parallel and perpendicular to the triad axis were determined from the slopes of the lattice parameter–temperature ((c, T) and (a, T)) curves. The results are shown in figs. 3 (*a*) and 3 (*b*). α_c refers to the expansion coefficient parallel to the triad axis, and α_a to the expansion coefficient perpendicular to this axis. α_c falls gradually from room temperature to 670° C, and then rises almost discontinuously to nearly twice its value. It then falls again from 700° C to 950° C. The behaviour of α_a is less noteworthy; it simply rises to a maximum at approximately 675° C.

These results are to be contrasted with those obtained by Chaudron and Forestier (1924) on polycrystalline compressed blocks of the oxide powder. Chaudron observed a maximum, rather than a discontinuity, in the (α, T) curve, and his (α, T) curve was not reversible but showed considerable thermal hysteresis. The differences between our curves and those given by Chaudron are not surprising in view of the fact that the X-ray measurements give the precise behaviour of the structure cell. The dilatometric measurements on the oxide powder are influenced by the state of subdivision of the powder, and can disclose no information concerning the anisotropic behaviour of the lattice.

§ 4. DISCUSSION

Our results indicate that the rhombohedral structure cell of haematite undergoes an abnormal change on cooling through the critical temperature of 675° C. The change comprises a sudden relative expansion along the triad axis, involving a change of the rhombohedral angle from the approximately constant value of 55° 24′ above the critical temperature to a value which decreases with decreasing temperature below 675° C, and attains a value of 55° 16′ at room temperature (see table).

Crystal Structure Data for α-Fe_2O_3

	Hexagonal system			Rhombohedral system	
Temp (°C)	a_H (Å)	c_H (Å)	c/a	a_R (Å)	α
20	5·0345	13·749	2·7311	5·4271	55° 15·8′
750	5·0856	13·845	2·7223	5·4698	55° 24·3′

This behaviour is analogous to that of the other iron oxides, FeO and Fe_3O_4, which undergo slight structural changes from cubic to rhombohedral symmetry by a relative expansion along a triad axis on cooling through certain temperatures. In the case of FeO the deformation first appears below −80° C (Rooksby and Tombs 1951), which is the critical temperature for the appearance of antiferromagnetism in that material. Fe_3O_4 exhibits the deformation (Tombs and Rooksby 1951) below the antiferromagnetic critical temperature of −160° C.

In α-Fe_2O_3 the atomic arrangement perpendicular to the triad axis resembles that perpendicular to a triad axis in the cubic structures of FeO and Fe_3O_4. In particular, sheets of iron atoms alternate with sheets of oxygen atoms, although the iron atom sheets are puckered in α-Fe_2O_3, and in FeO and Fe_3O_4 the oxygen atoms are in cubic instead of hexagonal close packing. Thus all three structures exhibit the same kind of abnormal lattice dilatation, in which the spacing between the iron atom sheets suddenly increases on cooling through certain critical temperatures of magnetic transformations. As the magnetic transformation involves the onset of the antiferromagnetic condition in FeO and Fe_3O_4, it appears that the structure cell behaviour of α-Fe_2O_3 is consistent with the view that haematite is also an antiferromagnetic oxide, with a critical temperature at approximately 675° c. The expansion coefficient–temperature curves in figs. 3 (*a*) and 3 (*b*) indicate that the exchange forces, which lead to the antiferromagnetic orientation of the electron spins, are directed mainly along the triad axis direction. This is in general agreement with the views of Néel (1949) and Shull *et al.* (1951) that the electron spins are parallel within a given iron atom sheet and antiparallel with respect to neighbouring sheets.

There is a close resemblance between the present observations on haematite and those made by Greenwald (1951) on the isomorphous chromium sesquioxide Cr_2O_3. In the case of Cr_2O_3, however, the dilatation on cooling through the antiferromagnetic critical temperature of 45° c comprises a relative contraction instead of expansion along the triad axis direction.

The existence of a lower transition point in haematite at −70° c has recently been reported by Guillaud (1951), who has shown that certain magnetic and thermal expansion abnormalities occur at this temperature. Shull *et al.* (1951) have suggested, on the basis of neutron diffraction observations, that a 90° rotation of the magnetic dipoles occurs at this transition point. Apart from single observations at −180° c and −70° c we have not studied the behaviour of the structure cell below room temperature. These observations showed, however, that there is no phase change between room temperature and −180° c, and that the rhombohedral angle is approximately the same at −180° c, −70° c and room temperature. In combination with the conception of a 90° rotation of magnetic dipoles this indicates that the exchange forces between the dipoles are independent of their orientation.

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The Thermal Oxidation of Tin

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Communicated by G. I. Finch; *MS. received 17th September* 1952

ABSTRACT. Electron diffraction experiments show that when tin foil is thermally oxidized in air an amorphous oxide layer is formed at temperatures up to 130°c. Above 130°c only crystalline SnO and SnO_2 are formed.

In 1932 Bragg and Darbyshire found that tin foil heated in a gas flame yielded electron diffraction patterns characteristic of tetragonal stannic oxide. It was noted by Steinheil (1934) that tin foil heated in air generally yielded electron diffraction patterns of stannic oxide, but in some cases diffraction patterns of stannous oxide were obtained. Jenkins (1935) found that films formed just above the melting point of tin consisted almost entirely of stannous oxide, but films formed at a higher temperature were composed of thedioxide SnO_2.

Recent experiments in this laboratory by Leu (1951) have established that an amorphous oxide film is formed on electropolished (001), (110) and (111) copper faces and heated in air at temperatures up to 130–140° c. Above this temperature crystalline Cu_2O films were formed. The experiments described below were carried out to see whether such amorphous oxide layers were formed also on tin at the lower temperature range.

Thinned tin foil was heated in air with a locally applied micro-gas flame for several minutes and then explored with an electron beam traversed through the heated region. The resulting electron diffraction patterns revealed that there was a gradual transition on passing from the unheated metal towards the most highly heated region from the initial tin (fig. 1, Plate), through an amorphous oxide (fig. 2) in the cooler regions, and a crystalline SnO stage (fig. 3) to the higher oxide SnO_2 (fig. 4) formed in the hottest region. Several faint rings visible in the haloes indicate that there was also a small amount of crystalline material formed together with the amorphous oxide.

Further specimens of tin foil were heated in a current of hot air for 15 minutes and it was found that the amorphous films were formed at temperatures up to approximately 130° c. In these cases the halo patterns were completely free of any sharp diffractions indicative of the presence of crystals.

These experiments show that the nature of the oxide layer formed on tin is dependent on the temperature at which it is formed, the fact of greatest interest being that the formation of a protective amorphous oxide on tin with rising temperature precedes the growth of the crystalline oxide.

ACKNOWLEDGMENTS

The author is indebted to Professor G. I. Finch for his constant interest and his greatly appreciated advice during the progress of this research.

The author also wishes to thank Dr. H. Wilman for his suggestions and helpful discussions.

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Accommodation Kinking Associated with the Twinning of Zinc

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MS. received 19*th June* 1952

ABSTRACT. Small changes of slope on cleavage surfaces of zinc single crystals have been measured by a stylus type profile recorder (Talysurf) and compared with photomicrographs of the same areas. Lattice bending through angles up to approximately 0° 47′ which relieves the localized shearing stresses due to twinning has been observed.

Other examples show firstly one twin with two associated accommodation kinks, and secondly a twin and its associated kink band occurring within the kink band associated with a second parallel twin.

Many of the larger scale effects of deformation twinning in zinc can be interpreted in terms of a simple shear in the $10\bar{1}2$ plane through an angle of about 8°, although, as was first recognized by Mathewson and Phillips (1927) the atomic movements are rather more complex. Since deformation is usually inhomogeneous, the resulting twinned zone is often irregular in shape. Thus it frequently assumes a lenticular cross section and may only extend for a relatively short distance through the crystal. When a crystal contains such a well-defined zone which has been sheared through a definite angle and which does not extend to the limit of the crystal, then there must be considerable strain in the remainder of the lattice.

In a recent paper Jillson (1950) has described how the strain may be relieved by accommodation kinking. It appears that at a distance from the twin which is large in comparison with its width, a definite lattice bend or kink, parallel with the twin, occurs and the stress is thereby concentrated in the volume of crystal between the kink and the twin. One of the ways in which this can occur is shown diagrammatically in fig. 1. These effects are particularly noticeable on a cleaved zinc surface where the twin plane makes an angle of 46° 59′ with the basal plane (cleavage plane) and the shear due to twinning produces a 3° 58′ change of slope in the cleavage plane. Due to the high reflectivity and flatness of the cleaved surface, the slight change of slope at the accommodation kink can be easily observed optically. The purpose of this note is to provide some direct visual evidence of some of the types of bending, and to point out that the angle of kinking can be simply measured by the use of a surface contour measuring instrument such as the 'Talysurf'.

Single crystals of 99·998% zinc of 1 cm^2 cross section were made by the Bridgman method in a graphite mould after sealing in glass under vacuum. They were cleaved while immersed in liquid air by a tensile stress normal to the cleavage plane. On some specimens where a partial cleavage had taken place in a nearby parallel plane the resulting bending stress had caused twinning. For examination of the accommodation effects a Vickers Projection Microscope was used. It was found that the best conditions of illumination and of lens aperture for showing the very small changes of slope in the cleavage plane were restricted with this particular instrument to the 33 mm lens and the plane glass illuminator which gave much better contrast than any other combination. At high magnifications the phase contrast equipment could sometimes be used but it was less reliable.

Figure 2 (see Plate) shows a cleaved surface with a twin (dark band) and the associated accommodation band. A Talysurf surface contour record taken across the same area is shown in fig. 3. In all examples of these records, the middle of the twin is shown by T and the accommodation band by K. The fine lines covering the whole of the field in fig. 2 are abrupt cleavage steps and it is apparent from the contour record which clearly shows the changes of slope associated with twinning, that the steps are only a small fraction of the height of the twin. The slope of the twin and the accommodation band can be measured. The Talysurf record gives the changes in height and the widths of the sloping parts can be more conveniently determined from photomicrographs. Seven determinations were made. The mean slope for the twin was 3° 55′ ± 9′ which compares favourably with the theoretical value of 3° 58′. The mean slope for the accommodation zone was 0° 47′ ± 2·5′.

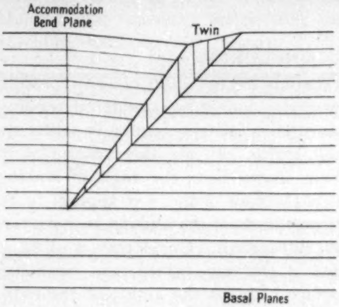

Fig. 1. Diagrammatic representation of lattice accommodation caused by twinning (after Jilson).

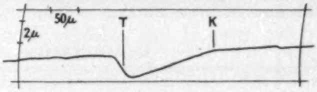

Fig. 3. Talysurf record across the twin shown in fig. 2.

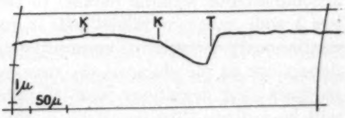

Fig. 5. Talysurf record across the twin shown in fig. 4.

Figure 4 (Plate) shows another twin which for part of its length has two accommodation bands. The contour record (fig. 5) shows two kinks accompanying the twin and for sixteen determinations the mean slope of the twin was 4° 24′ ± 21′. This deviation from the theoretical value is probably because the twin was narrower and an accurate photographic measurement could not be made. The precise position of the twin boundaries appeared to depend on the type of illumination used. However, the kink zones are many times wider than the twin and the error is proportionately smaller. The slope of the kink zone nearest the twin varied between 0° 42′ and about 0° 30′ while the slope of the zone furthest from the twin varied between 0° 14′ and about 0° 21′. The actual values appeared to depend on the position along the twin where the contour record was taken. This effect which is shown in fig. 6 suggests that a different accommodation kink has formed at each end of the twin and that in the area shown both kinks contribute to the accommodation. It was not possible to observe the ends of the twin because of oxide contamination.

A phase-contrast photograph of two parallel twins with two kink regions is shown in fig. 7 (Plate). The scratches made by the Talysurf needle can be seen and fig. 8 is the record from one of these. The slopes of the twins are 4° 16′ and 3° 32′ respectively while the slopes of the kinked zones are about 0° 22′ and 0° 12′.

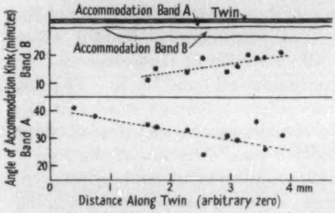

Fig. 6. The angle of the accommodation kinks shown in fig. 4 plotted as a function of their position along the twin.

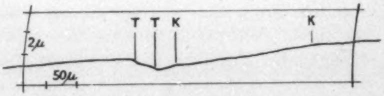

Fig. 8. Talysurf record across the twins shown in fig. 7.

DISCUSSION AND CONCLUSION

These experiments show clearly that, except in the rare cases of a twin passing uniformly through the specimen, the deformation cannot be considered as being localized in the twin, as additional deformation always occurs in order to accommodate the strain in the untwinned lattice. In the three examples of accommodation kinking parallel to the twin shown here, the angle of kinking has a wide range of values and, in fact, in the case of fig. 4 it appears to be continuously variable. The position and magnitude of the kinking is, however, dependent on the shape of the twin below the surface. Consideration of fig. 1 suggests that deviations from straightness of the sub-surface twin boundary will be indicated by curvature in the surface between the kink and the twin. Therefore a contour measuring method such as is used here will give information about the shape of the twin. These effects are being studied in more detail.

ACKNOWLEDGEMENTS

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Transistor Action in Lead Telluride

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MS. received 21*st August* 1952

ABSTRACT. Transistor action has been observed in single crystals of lead telluride, examined at 90° K. Properties of the units are described and it is found that values of current gain greater than unity may be observed.

§ 1. INTRODUCTION

The discovery of transistor action (Bardeen and Brattain 1948) and related phenomena has provided powerful methods for the examination of the electrical properties of solids. In the case of germanium and silicon fundamental constants such as carrier mobility and lifetime have been determined to a high degree of accuracy by the use of methods involving transistor action and the concept of carrier injection. Thus the occurrence of the transistor effect in any material is of importance since, even though a useful electronic device may not be immediately forthcoming, the application of methods involving carrier injection may enable more fundamental information concerning the particular material to be obtained. Furthermore, the occurrence of the effect in materials other than germanium and silicon indicates that the appropriate theories of transistor action must be extended so as to apply to these other materials.

The criteria for transistor action to occur in a semiconducting solid are that the material should be of relatively high purity, have a high carrier mobility and a narrow forbidden band. These criteria were applied to PbS and experiments showed that transistor action took place in suitable crystals of this material (Gebbie, Banbury and Hogarth 1950, Banbury, Gebbie and Hogarth 1951). In view of the many similarities in properties between the compounds PbS, PbSe and PbTe it was to be expected that transistor action would occur in PbSe and PbTe, and successful experiments with PbSe single crystals have already been reported (Hogarth 1951). In this paper the results of measurements on PbTe crystals which exhibit transistor action are described.

§ 2. EXPERIMENTAL METHODS

The crystals employed were artificial single crystals of p-type PbTe prepared by Lawson (1951, 1952) of this Establishment. Experiments on PbTe rectifier photocells carried out by Gibson (1952) together with the well-known results for PbTe photoconductive layers suggested that there was little or no indication of the existence of effective barrier layers on PbTe at or near room temperature. Since the transistor effect was known to be associated with the presence of barrier layers the experiments were carried out with the crystal assembly at a temperature of 90° K. The effect was observed in several specimens at this temperature, the results given in this paper being those for two crystals (specimens 1 and 2) whose conduction properties at 290° K were as follows:

Specimen	1	2
Conductivity σ (ohm^{-1} cm^{-1})	12	2·9
Hall constant R (cm^3/coulomb)	+84	+136
Carrier concentration (cm^{-3})	$8{\cdot}8 \times 10^{16}$	$5{\cdot}4 \times 10^{16}$
Mobility product $R\sigma$ (cm^2/volt sec)	1000	400

From experiments on similar specimens carried out by Putley (1952) it was expected that the Hall mobilities of holes and electrons at 90° K would be of order 10000 and 20000 cm^2/volt sec respectively.

The crystal holder and test-cell is shown in fig. 1. The small micromanipulator consisted of a brass plate on which was mounted an adjustable crystal holder and two holders for contact whiskers mounted on universal joints. Thus in making a transistor assembly the coarse manipulation could be carried out by means of tweezers and the final adjustment by means of the micromanipulator. The cell was then evacuated, cooled with liquid oxygen, and tested for contact properties. The whisker contacts were made of 0·004 in. phosphor-bronze pointed electrolytically, and the spacing between them was about 0·01 cm.

The electrical properties were investigated in the following manner. A test rack (of the type developed at Reading University and used by Banbury, Gebbie and Hogarth (1951)) was constructed, and contained bias supplies, variable a.c. input 50 c/s signals, and arrangements for inserting the appropriate variable resistances. The PbTe assembly could be connected directly to this and the voltages at various parts of the circuit monitored directly on the plates of a cathode-ray oscilloscope, any necessary amplification being provided via d.c. amplifiers. Currents were measured by monitoring the voltages developed across 22 ohm resistors. Thus characteristic curves relating emitter and collector currents and voltages could be displayed and recorded photographically. In the same way, the characteristic curves for each contact acting separately as a diode could be displayed and measured. A typical test circuit for the investigation of voltage gain of a PbTe transistor is given in fig. 2.

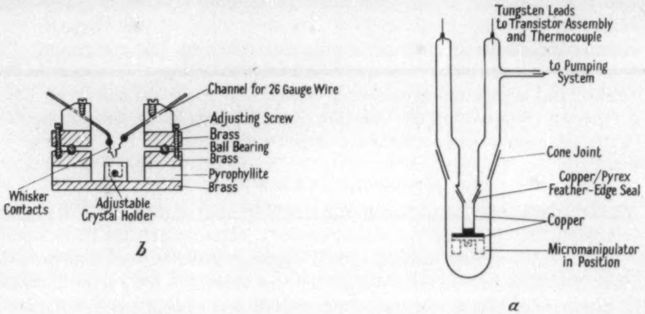

Channel for 26 Gauge Wire
Adjusting Screw
Brass
Ball Bearing
Brass
Pyrophyllite
Brass
Whisker Contacts
Adjustable Crystal Holder
b
Tungsten Leads to Transistor Assembly and Thermocouple
to Pumping System
Cone Joint
Copper/Pyrex Feather-Edge Seal
Copper
Micromanipulator in Position
a

Fig. 1. (*a*) Test-cell assembly and (*b*) Manipulator, used for experiments on transistor action in lead telluride. Note: the common surfaces of the micromanipulator and support table were ground together with carborundum before bolting together so as to ensure good thermal contact.

§ 3. EXPERIMENTAL RESULTS

All the crystals examined showed little or no rectification at 290° K and, as expected did not exhibit transistor action.

At 90° K the rectification characteristics were of the kind observed with p-type PbS at 290° K (Henisch and Granville 1951). The results for one assembly using cleaved surfaces of specimen 2 are given as follows. Figure 3 shows the curves relating voltage and current for emitter and collector contacts acting as diodes. These curves were measured after an electroforming treatment involving a 20-volt peak recurrent saw-tooth discharge through the emitter and collector in series, with the base on open circuit, had been completed. This treatment was given with the crystal at 90° K. Values of forward and reverse resistances in ohms were:

	Before forming		After forming	
	Forward at 0·5 v	Reverse at 3 v	Forward at 0·5 v	Reverse at 3 v
Emitter	100	1000	67	1000
Collector	900	50000	830	40000

The emitter characteristics were measured with 500 ohms in series and the collector characteristics with 10 000 ohms in series.

Curves relating collector and emitter voltages under conditions of constant collector current are given in fig. 4. It may be noted that voltage gains as high as 13 are obtained under suitable operating conditions. It may be further observed from fig. 5 (*a*) that values of current gain α in excess of unity occur. This phenomenon could not be observed before the electroforming process. Values of α as high as 3 have been observed by this means.

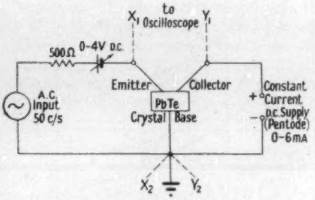


Fig. 2. Circuit used for the investigation of voltage gain in lead telluride.

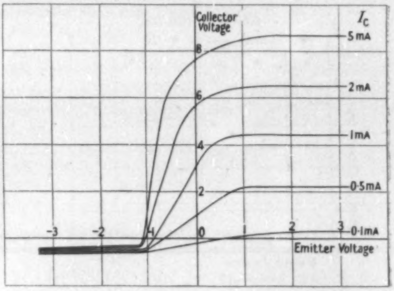


Fig. 4. PbTe transistor. Curves relating collector and emitter voltages. (Collector operated by a constant d.c. source, measured at 90° K).

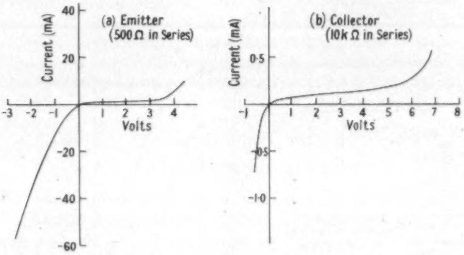
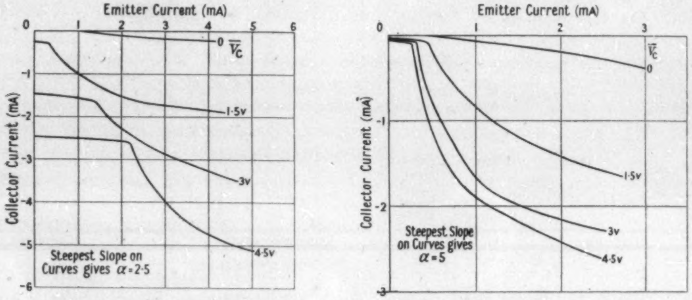


Fig. 3. PbTe transistor. (*a*) emitter, (*b*) collector, contact acting as a diode (measured at 90° K).

Figure 5 (*b*) shows a family of curves relating collector and emitter currents for a transistor assembly using specimen 1. Again values of α greater than unity may be noted. In this case forming was brought about by the momentary application of a 9-volt reverse d.c. bias in the collector circuit in the absence of a load resistance. This reduced the voltage gain to unity but the current gain increased as indicated, values of α as high as 5 being obtained.

It was found that electroforming the emitter contact by the method previously described for PbS (Banbury, Gebbie and Hogarth 1951) could also be achieved for PbTe, provided that the process was carried out at low temperatures.

The frequency response of voltage gain was measured using a low inductance circuit, a 0·1-volt input signal and a valve voltmeter for the measurement of output voltage between collector contact and earth. Two experiments were carried out using specimen 2, the point spacing being different in each case. The results are shown in fig. 6. As expected the voltage gain is larger and the cut-off frequency higher for the transistor with the smaller spacing between the contacts. The form of curve obtained is very similar to those obtained with Ge and PbS transistors (Banbury, Gebbie and Hogarth 1951), the voltage starting to decrease markedly at 100 kc/s and the output tending to zero at approximately 1 Mc/s.



(a) (b)

Fig. 5. PbTe transistors. Curves relating collector and emitter currents for various collector supply voltages (measured at 90° K). (a) Formed by saw-tooth discharge (emitter bias 1 v, series resistance 500 Ω). (b) Formed by momentary overloading of the collector (emitter bias 3 v, series resistance 500 Ω).

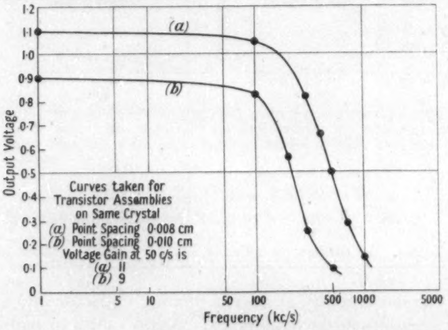


Fig. 6. PbTe transistors. Variation of voltage gain with frequency (collector load 10 kΩ ; collector bias 26 v ; emitter bias 1·5 v ; input voltage 0·1 v ; measured at 90° K).

§ 4. CONCLUSIONS

In view of the fact that good rectification and transistor action are only observed at low temperatures it seems unlikely that a useful device will be forthcoming from PbTe when connected in this manner. Nevertheless it is noteworthy that the predicted behaviour is in fact observed and that a further similarity in electrical properties exists between the photoconductors PbS, PbSe, and PbTe.

ACKNOWLEDGMENTS

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Electrode Phenomena in Transient Arcs

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ABSTRACT. High current, short duration arcs between various metals in air have been studied by means of a Kerr Cell camera and by observation of the marks left on the electrodes. Growth of these marks with arc duration is discussed and shown to be due to heat conduction in the case of the anode marks and to motion of the emitting areas in the case of cathode marks. Cathode spot temperatures are frequently too low for thermionic emission even on metals of high boiling point. At very high rates of current rise both anode and cathode marks are multiple.

§ 1. INTRODUCTION

When an arc strikes between two metallic electrodes, hot spots form on both anode and cathode within a very short time which is generally estimated to be of the order of 10^{-7} second or less. If the arc is prolonged the spots persist and most of the current is believed to pass through them. Cathode spots have been studied recently by Cobine and Gallagher (1948) and by Froome (1948, 1949, 1950) using techniques involving rapid motion of the spot across the cathode surface. The corresponding anode spots have received very little attention. When the arc has ended, a visible mark is usually left on each electrode. To avoid confusion we will use the term 'electrode marks' to denote these and will reserve the term 'electrode spots' to denote the active areas of the electrodes at any instant during the life of the arc.

We have studied the electrode marks left by transient arcs with durations ranging from one microsecond to one millisecond, struck in air at atmospheric pressure, between solid metallic electrodes. There was no general movement of the arc across the electrode surface, the centre of the arc being apparently stationary. In our experiments, arcs were studied under conditions of constant current, each arc being formed by the discharge of an artificial line, so that the current rose rapidly to a value which it retained for the remainder of the time during which the line was discharging. By using different lines, arc durations of 1, 5, 20, 200 and 1 200 microseconds could be obtained with currents ranging up to 200 amperes. The arc was usually struck by moving the electrodes together until sparking occurred, the potential to which the line was charged varying from 1 000 to 5 000 v, depending on the current required. Observations of the marks left when the arc was struck by suddenly connecting the charged line to the electrodes at different electrode separations showed that, unless it was very small, the electrode separation had no great effect on the nature of the marks. Numerous metals were used as electrodes, special attention being given to tin which has, from our point of view, the advantages of a low melting point (232° c) and a relatively high boiling point (2 270° c). Examination, on a cathode-ray oscillograph, of the instantaneous potential across the gap showed it to be from 20 to 30 v, so that the discharge may be appropriately described as an arc.

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We have examined microscopically the electrode marks left by these arcs. Such an examination is not an ideal means of obtaining information about the state of the electrode during the life of the arc because thermal processes such as melting and subsequent solidification take place after the arc itself has ceased, and one has the task of inferring an intermediate state from observation of the final marking. Nevertheless, we have found that something may be learned from this approach.

We have also used a Kerr Cell camera to obtain photographs of the arc at different times during its life. This camera was capable of taking three consecutive photographs with variable time intervals between the exposures. So that the three images would not be superposed, the light from the arc was split into three beams, each of which passed through a separate Kerr Cell shutter and formed a separate image on the film.

§2. THE ANODE

Kerr Cell photographs taken of arcs between a fine wire cathode and the plane surface of a tin anode show a bright glow near the anode which defines the anode 'spot'. Such photographs, taken at various intervals after the initiation of the arc show that the diameter of the anode spot remains approximately constant as the arc grows older. In the case of an 80-amp arc, for example, this diameter is 0·03 cm, so that, assuming that all the current passes through the luminous spot, the average current density is of the order of 10^4 amp/cm^2. The area of the anode spot increases with the current.

Microscopic examination of marks left on the anode by arcs with a moderate rate of rise of current shows that a single circular region has been affected by the arc. For most metals this region appears to have been molten, but a few metals (e.g. copper, tungsten) show only a discoloration of the surface. This latter group of metals also yields anode marks which show signs of melting if the surface is lightly oxidized or otherwise contaminated. Figure 1 (for all figures see Plate) shows a number of typical anode marks. For a given metal and arc duration, the mark area is approximately proportional to current.

For metals on which the anode marks have been molten, the melted area has a regular circular boundary and increases both with arc current and arc duration. Marks left on tin anodes by 80 amp 50 μsec arcs have radii a little over twice those of the corresponding spots. It appears that the increase of the molten area with arc duration is due to melting around a central active spot. The depth to which melting occurs can be found by using as anode two pieces of tin tightly clamped together, the arc being struck where the two pieces join, welding them together at this point. On separation the depth to which melting has taken place can be clearly seen. (Figure 2 shows the melting down of the surface of contact of the two pieces of tin which have been subsequently broken apart.) For 50 amp 200 μsec arcs the tin is melted to a depth of a little more than one-third of the radius of the anode mark. Depth of melting may also be studied by striking arcs on metal foil anodes of various thicknesses and observing the maximum foil thickness for which melting is apparent on the reverse side. Under these circumstances, where heat flow out of the back of the foil is largely prevented, the depth of melting is about twice as great as on a thick anode. Our observation of the appearance and behaviour of these anode marks is consistent with the belief that they are caused by the heating of the anode through electron bombardment which is concentrated over an area a few tenths of a millimetre in diameter.

On polished copper anodes, the anode mark is a discoloured area, probably consisting of oxides, which increases with arc duration. Kerr Cell photographs of the arc do not show a bright anode spot, but the appearance of the column near the anode suggests that the active area of the electrode does not vary much with time. The growth of the anode mark with time is probably due to oxidation around the active area.

The area and general appearance of the anode mark is independent of the material and shape of the cathode, and also of the inter-electrode distance, unless this is less than a few tenths of a millimetre. At these short distances the change in appearance is due to ejection of liquid metal from the molten area by the expanding air confined between the electrodes. If a 'Cellophane' screen is placed near the anode and a small hole is bored in it to allow the arc to strike, similar ejection of the anode metal occurs.

Anode marks left by arcs with a very high rate of current rise are usually multiple and are discussed later in this paper.

§ 3. THE CATHODE

Cathode marks are approximately the same size as anode marks and, like them, may or may not show signs of melting. In some other respects they differ from anode marks; their shape is usually less circular, their boundary more irregular, and their surface less smooth. Their appearance depends somewhat on the distance of the anode from the cathode, though the area is not altered significantly. In a previous publication two of us (Somerville and Blevin 1949) showed that the area of the cathode mark increased with arc duration and consequently that the apparent cathode current density, obtained by dividing the current by this area, decreased with time. We will now discuss whether this decrease is real or only apparent.

It is possible to advance several theories to account for the increase in the cathode mark area with arc duration. One which immediately suggests itself is that, when the mark has been molten, the increase in size may be due to progressive melting around a smaller active centre, as appears to be the case with anode marks. This explanation may be discarded because it is found that marks left on thin foil cathodes may have diameters ten or more times the thickness of the foil, without any sign of melting appearing on the reverse side. Clearly, if the large mark were due to melting from a small central spot, melting right through the foil would have taken place long before it had extended so far laterally. Moreover, the irregular appearance of many marks is inconsistent with melting from a centre.

It might also be thought that the increase in mark area on low melting point cathodes could be due to heat generated by dissipation of electrical energy in the resistance of the cathode material. However, if the same current pulse is led into the cathode by actual contact with a very fine copper wire whose diameter is much less than that of the observed cathode mark, it is found that no mark is left on the cathode at all, showing that the Joule heat generated is much too small to cause any melting. Calculation supports this view.

Clearly, in the case of the cathode materials with high melting points where no melting takes place, neither of these theories is applicable.

Holm (1946) has developed a theory which predicts an increase in the active spot area with time. He assumes the whole active area to be maintained at a temperature of 2000° c, and he calculates the area which the incoming energy, assumed delivered at a constant rate, is capable of keeping at that temperature. This area, which increases with time, he takes to be equal to the active spot area. We do not think that the whole area of the cathode mark attains a temperature as high as this at any one time during the first few hundred microseconds of life of the arc. Our reasons for this belief are as follows.

In the first place, melting is usually not apparent in cathode marks on clean metals with melting points above about 900° c. For example, the cathode marks of 50 amp 200 μsec arcs on tin, cadmium, lead, zinc, antimony and aluminium, whose melting points range from 232° c to 660° c, show definite but decreasing degrees of melting, while marks left by similar arcs on cathodes of silver, gold, copper, nickel, cobalt, platinum and tungsten, whose melting points range from 961° c to 3387° c, exhibit no melting at all, not even of a localized nature within the main mark area. The metals were found to be similarly grouped when marks left by 50 amp 20 μsec arcs were examined. When the surface is heavily oxidized or contaminated in other ways, melting may occur even on very high melting point metals such as tungsten.

Further information about the average cathode temperature at the centre of the mark may be obtained from a comparison of anode and cathode marks. It is found for tin that the depth of melting at the cathode, measured either by using foils or a split cathode, is about half the corresponding depth at the anode for a similar arc. Consequently one would expect that the average temperature at the cathode would be of the order of half that at the anode. As the anode temperature presumably cannot exceed the boiling point of tin, 2 270° c, the average cathode temperature would not be likely to exceed about 1 000° c. A comparison between cathode marks on tin and anode marks on cadmium is also instructive. These two metals have very similar thermal properties except for their boiling points, that of cadmium being 767° c (specific heats: 0·055, 0·054 cal g^{-1} (deg c)$^{-1}$; thermal conductivities: 0·22, 0·16 cal cm^{-1} sec^{-1} (deg c)$^{-1}$; latent heats of fusion: 14, 14·6 cal/g; melting points 321, 232° c for cadmium and tin respectively). Consequently heat flow and melting should proceed in a very similar manner in both. We find that tin cathode marks and cadmium anode marks have much the same diameter and the same depth of melting. This indicates that the average temperatures of the marks on each electrode were approximately equal, and therefore neither could have exceeded the boiling point of cadmium.

It is possible to go somewhat further and make an approximate calculation of the average surface temperature of the cathode at the centre of the mark, based on the observed depth of melting. Because this depth of melting is small compared with the spot diameter the problem approximates to that of linear heat flow into a semi-infinite metal solid whose infinite plane boundary is suddenly raised to, and maintained at, a temperature T_s greater than the melting point of the metal. If, as we shall see later, we are not justified in assuming the cathode mark area to be raised to a uniform temperature, then T_s represents an average surface temperature.

Lightfoot (1930) has given an exact solution to this problem, taking into account the energy absorbed as latent heat. Suppose that a semi-infinite metal at temperature T_0 extends from the plane $x=0$ to infinity in the positive direction of x, and that at time $t=0$ the plane $x=0$ is raised to a temperature T_s and kept at that temperature. Then with some changes in notation and a little algebra we find, from Lightfoot, that the temperature at time t and distance x $(<2\mu(\kappa t)^{1/2})$ is given by

$$T=T_s-(T_s-T_0)\operatorname{erf}\left(x/2(\kappa t)^{1/2}\right)-(L\mu\sqrt{\pi}/s)\operatorname{erf}\left(x/2(\kappa t)^{1/2}\right)(1-\operatorname{erf}\mu)\exp\mu^2 \qquad \ldots\ldots(1)$$

where L is the latent heat of fusion, s is the specific heat, and $\kappa=K/s\rho$ where K is the thermal conductivity and ρ the density of the metal. μ is a parameter. If ξ is the distance of the solid–liquid boundary from the plane $x=0$ at time t, then

$$\xi=2\mu(\kappa t)^{1/2}. \qquad \ldots\ldots(2)$$

Suppose now that the temperature of the metal is raised to T_s, only for a time τ corresponding to an arc of that duration. At the end of the arc the melted boundary will have reached $x=2\mu(\kappa t)^{1/2}$ but, because of the heat stored in the melted region, melting will continue to a greater depth after the arc has ended. It is not difficult to show from (1) that, during the time τ the heat entering the metal per unit area is

$$2(Ks\rho\tau/\pi)^{1/2}\{T_s-T_0+(L\mu\sqrt{\pi}/s)(1-\operatorname{erf}\mu)\exp\mu^2\}.$$

If *all* this heat went into melting the metal it would be melted to a depth d where

$$d=2(\kappa\tau/\pi)^{1/2}\{T_s-T_0+(L\mu\sqrt{\pi}/s)(1-\operatorname{erf}\mu)\exp\mu^2\}/(T_m-T_0+L/s), \qquad \ldots\ldots(3)$$

T_m being the melting point of the metal.

In a semi-infinite metal all this heat will not be used in melting metal, but if instead of the semi-infinite metal we have a thin foil which just melts through to the other side, then, because heat flow out of the back of the foil is largely prevented, all the heat entering will be used in melting.

Considering d as the thickness of this foil, (3) gives an equation connecting T_s and μ. Another equation connecting these two quantities may be obtained from (1) by putting $T=T_m$, in which case from (2) $x/2(\kappa t)^{1/2}=\mu$ and (1) becomes

$$T_s-T_0=(T_m-T_0)/(1-\operatorname{erf}\mu)+(L\mu\sqrt{\pi}/s)\exp\mu^2\operatorname{erf}\mu. \qquad \ldots\ldots(4)$$

Equations (3) and (4) can be solved simultaneously without great difficulty by computation or by graphical methods to give T_s and μ. The results of some such calculations, based on measured values of d obtained with tin foils, are given in the table. This shows that the temperature of the cathode surface near the centre of the mark, if constant, probably does not exceed 1 200° c. It will also be seen that the distance to which melting has reached by the end of the arc is approximately half the thickness d of the foil, so that the error in applying to a foil cathode the heat intake calculated for an infinitely thick cathode is probably not excessive.*

Tin Cathodes

τ (μsec)	d (cm)	μ	ξ (cm)	T_s (° c)
200	0·015	0·53	0·008	630
50	0·013	0·77	0·006	1160
20	0·008	0·73	0·0036	1090
5	0·0025	0·55	0·0013	670

We have just shown that if the temperature over the cathode mark is uniform, it is not likely to be high enough to give appreciable thermionic emission. It also seems unlikely that the current can be uniformly distributed over the whole area of the cathode mark because then the cathode current density of arcs lasting more than a few microseconds would be of the order of 10^4 to 10^5 amp/cm^2 which is much too low to give sufficient field emission (Mackeown 1929, Froome 1950, Wasserrab 1951). Our observations of the appearance of the cathode mark thus point to the conclusion that it must have had a fine structure during the life of the arc.

In order to determine whether this is so, photographs of the cathode region of the arc were taken with the Kerr Cell camera. Figure 3 shows two typical photographs of an arc struck in air between a thin copper wire anode and a plane tin cathode.† These photographs were taken obliquely, with the axis of the camera making an angle of about 20° with the plane of the cathode, and so the reflection of the arc column in the cathode can also be seen. Also included in the figure is a photomicrograph, to the same scale, of the cathode mark left by the arc in the second photograph. The growth of the luminous area with time is clearly shown, contrasting with the behaviour at the anode. In the second photograph several intense elements may be seen, apparently distributed round the periphery. These may well be emitting areas moving outward in a manner similar to those observed on liquid metallic surfaces by Froome (1949, 1950).‡ The peripheral appearance of the cathode mark in fig. 3 (*c*) rather supports the view that it was generated by several separate centres of activity moving radially outwards.

We have also struck arcs on the edge of thin foils held in a sandwich between two pieces of bakelite, with only the edge of the foil exposed. This has the advantage of allowing all parts of the emitting area to be held in sharp focus at once, which cannot be done with oblique photographs of arcs on a plane cathode. Photographs of such arcs on the edge of thin foils show that there are many small discrete emitting areas distributed along the length of the foil.

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* Our colleague Dr. R. C. T. Smith has obtained a more refined solution for the heat flow problem relating to arcs struck on thick electrodes. Calculations based on preliminary experiments show that the mean temperature at the centre of the cathode spot can be well below the boiling point in this case also.

† In order to obtain greater clarity for reproduction, these are single photographs of two similar arcs. They exhibit the same general characteristics as a sequence of photographs of a single arc.

‡ In a private communication some time ago Dr. Froome suggested to us that the increasing areas of our cathode marks with time might be explained in terms of the outward motion of line elements like those observed by him.

§ 4. MULTIPLE ELECTRODE MARKS

When the rate of rise of the arc current is very high the anode mark is usually multiple and may consist of fifty or more small distinct marks distributed over an approximately circular area (fig. 1 (*d*)). These small marks tend to coincide with small surface irregularities, such as scratches, and their number and distance apart increases as the rate of rise of the current increases. Multiplicity occurs when this rate exceeds about 10^7 to 10^8 amp/sec, a value similar to that obtained by Froome (1949, 1950) as the condition for multiplicity of cathode spots on liquid mercury and sodium-potassium alloy surfaces. However, anode mark multiplicity does not appear to be directly associated with multiplicity of the cathode mark. For a 150 amp 1 μsec arc on plane tin electrodes, the anode mark is highly multiple and spread over a relatively large total area, the cathode mark single and of small area. The apparently single cathode mark probably has a fine structure during the life of the arc, but the following experiment shows that a correspondence between cathode and anode mark fine structure is unlikely.

A 150 amp 1 μsec arc was struck between two tin electrodes about 3 mm apart. Between these electrodes two 'Cellophane' sheets were placed, parallel to the plates and distant approximately 2 mm from them and from each other. In each sheet there was a small hole about 0·03 mm in diameter, the line joining the two holes being perpendicular to the plates. The arc passed through these two holes. (The current density in the holes had the very high value of 2×10^7 amp/cm^2.) Both marks were multiple, but it seems unlikely that any discrete channels linking the small anode marks to possible corresponding marks on the cathode could pass together through the two small holes in the 'Cellophane' and still retain their identity. Apparently anode mark multiplicity is associated with processes at or near the anode. In a further experiment a single 'Cellophane' sheet with a similar hole was placed between cathode any anode and its distance from the anode varied. As it approached within a few tenths of a millimetre of the anode the area over which the small anode marks were spread diminished, suggesting that multiplicity is not confined entirely to processes occurring at the actual anode surface. It should perhaps be added that we have checked that the small anode marks are not due to an oscillatory discharge of the small interelectrode capacity nor to any brush discharge taking place prior to the main discharge.

Figure 4 shows a photograph of a 50 amp 5 μsec arc of comparatively low multiplicity on a tin anode, the shutter being open for the whole duration of the arc. Note the correspondence in position of the anode spots in (*a*) and the marks in (*b*). There is also a characteristic single bright core at the centre of the column.

Multiplicity of the cathode mark is not uncommon for arcs with a high rate of current rise. This is not unexpected in view of the general multiplicity of the cathode spot, and indeed absence of multiplicity is usually due only to melting together of the separate marks.

§ 5. THE INITIAL ELECTRODE TEMPERATURE

Arcs of various currents and durations have been struck on electrodes whose temperatures were held constant at points within the range -50° c to 220° c. The area of marks left on tin cathodes increased as the initial temperature was raised, the rate of increase becoming greater as the melting point (232° c) was approached. For example, a 50 amp $5\,\mu$sec arc struck on a tin cathode at 200° c leaves a mark about four times the area of that left on the same cathode at -50° c. Molten marks on oxidized copper cathodes showed a definite but much smaller increase through the same temperature range. Anode marks on these metals also increased with the temperature but to a much smaller degree.

§ 6. PRESSURE VARIATION

Arcs have been struck in air at pressures ranging from one atmosphere to 1 cm Hg. Marks on both electrodes were little different in appearance or area from those obtained at atmospheric pressure.

§ 7. CONCLUSION

Our experiments lead us to the following conclusions about high current, transient arcs in air.

(1) The active area at the anode does not increase with arc duration. Growth of the anode mark is due to heat conduction from this active centre.

(2) The cathode spot becomes multiple soon after the initiation of the arc, and the increase in mark area is due to an outward motion of these centres of emission. For arcs of duration less than a few hundred microseconds heat flow beyond the active area is not very important in mark formation.

(3) Even on some high boiling point metals, the temperature of the cathode spot does not exceed about 1200° c.

(4) Multiplicity at the anode occurs under similar conditions to those reported for cathode multiplicity ; that is, when the rate of rise of current is very high. It is possible to have multiplicity at one electrode only, and when it occurs on both there is no connecting fine structure in the arc column.

ACKNOWLEDGMENTS

We are greatly indebted to Dr. R. C. T. Smith for many very helpful discussions about heat flow problems associated with this work. The project has been generously supported by the Commonwealth Electrical Research Board.

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The Measurement of the Angular Diameter of Radio Stars

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ABSTRACT. A new method of using interferometers to measure the diameters of radio stars is described. The method is based on the use of the phase-switching system, and has several practical advantages over previous methods.

Measurements which have been made on the radio star (23.01) in Cassiopeia, indicate that the radio star has an angular diameter of about 5′ arc, the exact value depending on the distribution of brightness across the star. A gaseous nebula reported by Baade, in the same position as the radio star, has about the same diameter. Some measurements have also been made on the radio star (19.01) in Cygnus, and this star appears to have an angular diameter of about 3½′ arc.

§ 1. INTRODUCTION

THE angular diameter of the sources of radio waves associated with sunspots was first measured in 1946 (Ryle and Vonberg 1946, McCready, Pawsey and Payne-Scott 1947). The method employed was closely analogous to that which Michelson used for visual stars, since it depended on a measurement of the ratio between the maximum and minimum signals received by an interferometer aerial system. Attempts have been made to measure the diameters of radio stars by this method, but they have only given an upper limit for the diameter (Ryle and Smith 1948, Bolton and Stanley 1948, 1949, Stanley and Slee 1950).

A different method of using an interferometer for this measurement was recently suggested by Ryle (1952) and it is the purpose of this paper to consider in more detail the application of this method and to describe some measurements made with its use.

The new method is based on the phase-switching technique described by Ryle. If a radio star appears as a uniformly bright disc with an angular diameter $\Delta\theta$ the amplitude of the record obtained from a phase-switching interferometer will be proportional to $\gamma = J_1([\pi d/\lambda]\Delta\theta)/\{(\pi d/\lambda)\Delta\theta\}$ where d is the distance between the aerials and λ is the wavelength. This factor corresponds to the 'visibility' of the fringes in the optical analogy; in the original radio methods, where the total power received by an interferometer was recorded, the quantity measured was $(1+\gamma)/(1-\gamma)$.

The disadvantage of the original method lay in the determination of the minimum power from the source and in the practical difficulty of equalizing the gains of the two aerials and of the preamplifiers which might be necessary to overcome the losses in the transmission lines. Ryle suggested that γ could be measured with the phase-switching interferometer by observing the record amplitude obtained with the interferometer aerials at varying spacings. The background radiation would then not introduce difficulties in determining the minimum power, and it would no longer be necessary to equalize the gains; the gain of each aerial must however remain constant during the whole series of observations. This would be difficult since, during this time, one or both aerials (together with the associated preamplifiers and transmission lines) must be moved considerable distances.

A technique for measuring γ has recently been evolved which avoids these difficulties. In this method a single movable aerial is used, with a phase-switching recording apparatus, alternately in conjunction with each of two fixed aerials; it is now no longer necessary for the gain of the movable aerial to remain constant.

Before describing the new method it is important to examine the fundamental limitations which apply to all interferometric methods of diameter measurements. These limitations are discussed in the next section; the practical difficulties encountered in the original phase-switching method are discussed in § 3, and the manner in which they are avoided by the new method is indicated in § 4.

§ 2. FUNDAMENTAL LIMITATIONS IN THE MEASUREMENT OF ANGULAR DIAMETER BY INTERFEROMETRIC METHODS

Even if there were no practical difficulties of the type mentioned above, there are a number of factors which limit the smallest angular diameter which may be resolved.

(i) *Confusion by adjacent weak Radio Stars*

Since the width of the primary reception pattern of each aerial of an interferometer is large compared with the angular separation of the maxima of the interference pattern λ/d, the amplitude of the periodic trace on the record may be influenced by sources of radio waves situated over a comparatively large solid angle. If the star under observation were in a region where the background radiation could be regarded as uniform over regions of the order of λ/d, the observed periodic component can be assumed to be entirely due to the wanted star. If, however, the region exhibits irregularities of small angular extent, such as would occur in the presence of other faint radio stars, the observed amplitude may be modified by an amount which is related to the intensities of those stars; the diameter deduced from observations at two different aerial spacings may then be subject to serious errors, particularly when the wanted star is not of great intensity, or when the available aerial spacing is limited and γ does not differ greatly from unity.

The order of magnitude of the effect can be deduced from the known magnitude distribution of radio stars, and some examples are given in Appendix A, where it is also shown that this 'confusion' effect may be greatly reduced by making observations with a large number of different aerial spacings.

(ii) *Scintillation*

A further important limitation is caused by the 'scintillation' of radio stars, which has been shown to be due to random diffraction by ionospheric irregularities. It has been found that in addition to the random fluctuations of intensity, there are random variations of the apparent position of the star (Ryle and Hewish 1950). If the period of variation of the position is large compared with the response-time of the recording apparatus the effect is not serious, but if the period of variation is comparable with, or smaller than, the response-time, the record amplitude will be decreased; it will then be impossible to measure an angular diameter smaller than the angular deviations of the position of the source.

This effect may be particularly important at the lower frequencies, where the angular deviation is greater. At 80 Mc/s, for example, angular movements of about 2′ arc with a duration of a few seconds are frequently observed. There are, however, long periods especially in the daytime when the scintillation is very small (Ryle and Hewish 1950).

(iii) *Reading Accuracy and Output Time Constant*

The accuracy with which the amplitude of the record may be read is often limited by the random fluctuations on the record. These fluctuations may be reduced by using a large output time constant, but it must be remembered that the record amplitude itself will be reduced if the time constant becomes comparable with the periodicity of the record; the value of the time constant would then have to be known with considerable accuracy to correct the observed amplitudes at different aerial spacings.

§3. PRACTICAL DIFFICULTIES IN THE USE OF SPACED AERIAL INTERFEROMETERS

It has already been shown (Ryle 1952) that the use of the phase-switching system with different aerial spacings makes it possible to measure the angular diameter of a source without the necessity for equalizing the gains of the two aerials; this method also overcomes the difficulties associated with the measurement of the background radiation and hence the minimum power from the source.

It is now important to examine the remaining difficulties which may limit the resolving power which may be achieved; it will later be shown how these difficulties have been overcome in the new method. In order to appreciate these practical difficulties it is worth noting that for measurements of sources whose angular diameters are only about 5′ arc with interferometers having apertures of 100–200 λ, it is necessary to measure values of γ of about 0·95; the overall sensitivity of the recording system must therefore remain constant to within a few per cent for the duration of the experiment.

The deflection sensitivity of a phase-switching interferometer depends primarily on the following factors: (i) the geometric mean of the aerial gains, (ii) the losses in the transmission lines, and the characteristics of the preamplifiers used to offset these losses, (iii) the gain of the recording receiver. In most applications of the phase-switching system it is convenient to use on the recording receiver an automatic gain control circuit, whose function it is to maintain the total noise power at the output of the receiver at a fixed value. In this way the deflection becomes independent of the variations in the main receiver, and depends only on the predominating source of the noise power.

At the lower frequencies, where the general galactic radiation produces a power greater than that generated in the preamplifiers, the recording sensitivity when a particular part of the galaxy is being observed is proportional to a slowly varying function of the preamplifier gains $(\beta_1\beta_2)^{1/2}/(\beta_1+\beta_2)$ where β_1 and β_2 are the two gains.

At the higher frequencies where the preamplifier noise predominates, the recorder sensitivity also depends on the noise powers generated in the two preamplifiers.

In addition to the direct effects described above, further small effects may be produced by variations in the frequency response of the preamplifiers or by differences in the electrical lengths of the transmission paths from the aerials to the receiver. These effects are discussed in Appendix B.

It may therefore be seen that if values of γ of 0·95 were to be measured, it would be necessary to ensure that one of the aerials together with its preamplifier could be moved without the gain of the preamplifier changing by more than about 30% or its noise power by more than about 5%.

It has been found difficult to maintain these figures over a period sufficiently long for a complex experiment at a number of aerial spacings to be carried out, and the new method which does not require this long term stability will now be described.

§4. THE NEW PHASE-SWITCHING METHOD OF DIAMETER MEASUREMENT

In the new method the difficulty of maintaining a constant sensitivity over a long period is overcome without the necessity of measuring the sensitivity of the apparatus at any time. There are two aerials whose gains must remain constant, but these need not be moved. A third, movable, aerial is used in conjunction with these two. The ratio of the record amplitudes when the third aerial is used with each of the other two in turn is not dependent on the gain of the third aerial; as the position of this aerial is changed, so that the spacing of these two interferometer pairs is changed, the variation in this ratio depends only on the variation of the quantity γ.

The arrangement of interferometers used in the new method is illustrated in fig. 1.

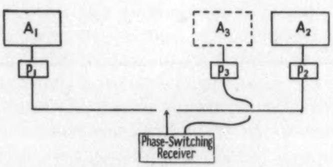

Fig. 1. Arrangement of interferometers for the measurement of angular diameter.

The aerials A_1 and A_2, with their preamplifiers p_1 and p_2, are fixed. They are used alternately in interferometers with the third movable aerial A_3, with preamplifier p_3. The phase-switching receiving system is then connected permanently to A_3, with either A_1 or A_2 connected to the other input.

During the transit of the radio star, the two combinations of aerials are used alternately for periods of a few minutes, while several cycles of the record are obtained. The ratio of the record amplitudes may then be calculated. This ratio is the ratio Γ of the values of γ for the two spacings A_1A_3 and A_2A_3, multiplied by a factor depending only on the characteristics of the two fixed aerials A_1 and A_2. The experiment may now be repeated with the common aerial at other positions, and comparison of the results will at once give the variation of Γ as the two spacings vary. From this variation of Γ it is possible to deduce the diameter of the star.

The observations might conveniently be begun with the movable aerial exactly halfway between the two fixed aerials. The two values of γ would then be the same, and the ratio of record amplitudes could be used to calculate Γ for all succeeding positions.

Leaving aside for the moment certain secondary effects of the preamplifiers, the characteristics of the movable aerial A_3 do not enter into this measurement, and it is in fact possible to use a series of fixed aerials, each with its own preamplifier and transmission line, instead of one movable aerial A_3. It might in this way be possible to use larger aerials than could conveniently be moved between observations. One method of doing this uses a symmetrical disposition of four aerials as shown in fig. 2, where the distances A_1 A_3' and A_3'' A_2 are equal.

Fig. 2. Disposition of aerials for direct measurement of Γ.

If aerials at A_3' and A_3'' are used successively as the third aerial, the ratio of record amplitudes is proportional first to Γ and then to $1/\Gamma$, so that Γ may be found directly.

Although the methods outlined above do not require that the fixed aerials A_1 and A_2 or their associated preamplifiers and transmission lines should be in any way similar, there are effects which may influence the measured values of Γ if they are different in certain respects. It is shown in Appendix B that differences in the frequencies to which the preamplifiers are tuned constitute a serious difficulty, but that if the preamplifiers on these two aerials are interchanged periodically, any differences which may arise during the experiment may be allowed for.

With a receiver of large frequency bandwidth, the true record amplitude will only be obtained when the electrical paths from the star to the receiver via the two aerials are approximately equal. It is therefore necessary firstly that the observations should be made when the star is near the collimation plane of the interferometer, and secondly that the three transmission lines should be approximately equal. It is particularly important that the electrical paths to the two fixed aerials A_1 and A_2 should be equal, since effects due to the receiver bandwidth will then be identical when the movable aerial A_3 is used with either A_1 or A_2.

§ 5. OBSERVATIONS OF THE DIAMETER OF TWO RADIO STARS

(i) *Observations on a Wavelength of* 3·7 *m*

The first observations of the radio star in Cassiopeia (23.01) using the phase-switching technique were made with $\lambda = 3{\cdot}7$ m. The large aerials of the interferometer previously used for observations of radio stars (Ryle, Smith and Elsmore 1950) are so constructed that the two halves of each aerial may be used independently. The electrical centres of these two parts are 5λ apart in the direction of the interferometer axis, so that the four parts of these aerials could be used in the 'four aerial' method for the direct measurement of Γ. The two spacings were approximately 110λ and 5λ.

With these aerials the probable limit of accuracy due to confusion, calculated according to Appendix A, was about 2%. A value of $\Gamma = 0{\cdot}97$ was obtained in this experiment, and it was decided to use a higher resolving power in an attempt to obtain a more significant result.

(ii) *Observations on a Wavelength of* 1·43 *m*

Five different aerials were used with $\lambda = 1{\cdot}43$ m in a series of experiments to determine the angular diameters of the radio stars in Cassiopeia (23.01) and Cygnus (19.01). Two of these were the parabolic reflector aerials with a diameter of 6λ and a spacing of 200λ which were used for the accurate location of radio stars (Smith 1951). Two more were identical broadside arrays of five full-wave dipoles mounted above a reflecting sheet. The fifth was similar to the last two, and was used as a movable aerial with either of the two pairs.

The attenuation of the transmission lines, which were all arranged to be equal in length within a few wavelengths in any particular experiment, was between 6 db and 12 db according to the length used. Preamplifiers with a gain of 14 db and an input noise temperature of about 1500° K were used. The noise power at the receiver input originated mainly in the preamplifiers, and it was important to interchange the preamplifiers on aerials A_1 and A_2 at intervals. Observations over a period of 7 days showed that in this time variations in preamplifier characteristics caused a slow drift of 5% in the observed ratios, this amount being detected by interchanging the preamplifiers.

With these aerials, the limit of accuracy due to confusion was of the order of 5% in a single experiment, but since a movable aerial was used it was possible to use several positions for observations where γ was greater than 0·9, so that a significant result could be obtained.

The output time constant of the recording apparatus was 2 seconds, and only small corrections were necessary for the resultant reduction in record amplitude. Spacings of up to 430λ were used giving a minimum lobe separation of 8′ arc.

Observations of the radio star 23.01 (*Cassiopeia*). Table 1 gives the measured values of Γ, with their estimated accuracies, at various distances between the aerials of each interferometer pair A_1A_3 and A_2A_3.

Table 1

Distance (metres)		Values of Γ (radio star 23.01)	
A_1A_3	A_2A_3	Measured	Calculated (5·5′ diam.)
53	605	$0{\cdot}52 \pm 0{\cdot}05$	0·52
187	472	$0{\cdot}70 \pm 0{\cdot}02$	0·72
52	334	* $0{\cdot}81 \pm 0{\cdot}02$ $0{\cdot}87 \pm 0{\cdot}03$	0·83
18	266	$0{\cdot}92 \pm 0{\cdot}03$	0·89
108	177	$0{\cdot}97 \pm 0{\cdot}02$	0·97

* These different values were obtained in two separate experiments using different aerial systems.

The values of Γ given in the last column are those which would be expected from a uniformly bright circular disc of 5·55′ arc diameter. The correspondence of these figures with the observed values of Γ show that a source of this size and shape could account for the observations. The accuracy in size is within about $\pm 0{\cdot}2'$ arc.

Observations of the radio star 19.01 (*Cygnus*). Some observations were also made on the radio star 19.01 using a wavelength of 1·43 m. The same arrangement of aerials was used as for the observations on the star 23.01. The results are shown in table 2.

The calculated values of Γ are those expected from a uniformly bright disc of 3·6′ arc diameter. The measurements are less accurate than those for the star 23.01 because fewer observations were made; the results in table 2 indicate that the accuracy is within about ± 0·3′ arc.

Table 2

Distance (metres)		Values of Γ (radio star 19.01)	
A_1A_3	A_2A_3	Measured	Calculated (3·6′ diam.)
53	605	0·79 ± 0·05	0·76
187	472	0·83 ± 0·02	0·85
52	334	0·94 ± 0·03	0·93

§6. THE IDENTIFICATION OF THE RADIO STAR (23.01)

In a private communication, Dr. W. Baade of Mt. Wilson and Palomar Observatories has stated that a nebula has been discovered at the position of the radio star 23.01 (Smith 1951). This nebula appears as a roughly circular area, 5′ arc in diameter, in which there are numerous luminous patches. The centre of the area corresponds closely with the position of the radio star.

The correspondence between the angular diameter of this object and the results of the measurements just described appears to be highly significant. If the region is 'optically thick' for the radio observations, it might appear as a uniformly illuminated disc only slightly smaller than the radio source. Alternatively the region might be 'optically thin' for radio wavelengths, and the appearance would then depend on the distribution of the radiating gas in the sphere. For example, a thin shell of 5′ arc in diameter would give the same results in these measurements as a uniform disc about 5½′ diameter, and a sphere of the same diameter but of constant density would resemble a uniform disc of about 6½′ diameter. The measurements favour the first two possibilities rather than the third. The assumption that the emitting region is confined to the area observed by Baade may not be justified. If, for example, the source is more extended, with a surface brightness decreasing radially according to a gaussian function, the brightness would have to fall to half the central value on a circle of about 4′ arc diameter. The form of the variation of γ with aerial spacing should show significant differences between these various models if measurements could be made at considerably greater aerial spacing.

§7. THE BRIGHTNESS TEMPERATURE OF THE RADIO STARS

The intensity of the radiation from a source of radio waves may be expressed at a given frequency in terms of the 'brightness temperature' of the emitting area. This may be defined as the temperature of a black-body radiator which would radiate with the same intensity at that frequency. The brightness temperature may of course vary across the surface of the emitting region.

Although the precise distribution of brightness across the two radio stars has not yet been determined, it is possible on the basis of the present results to give an approximate figure for the mean brightness temperature over the disc. The flux density from these stars has been determined by Ryle, Smith and Elsmore (1950) at 81·5 Mc/s, and recent work has shown that the intensities of both stars on 210 Mc/s conform to these values assuming the spectral distribution (flux density proportional to λ) determined by Stanley and Slee (1950) for the star 19.01 (Cygnus A). Assuming that the diameters of the stars are those given in § 6, the brightness temperature of both stars is about 2×10^7 deg K at 81·5 Mc/s, and 10^6 deg K at 210 Mc/s.

§ 8. CONCLUSIONS

The new method of using interferometers for measuring angular diameters of radio stars has shown that the radio stars 23.01 and 19.01 have measurable angular diameters. The results of the present series of observations are not consistent with measurements of the upper limit of the angular width of the radio star 19.01 by Stanley and Slee (1950). Observations by Mills and Thomas (1951) of the scintillation of this star also suggest that the diameter is less than 3′ arc.

The radio star 23.01 may possibly be identified with a gaseous nebula, whose position and diameter correspond closely with the measured position and with the measured diameter of 5·5′ arc.

From these results and the observed intensities, it is possible to deduce the 'brightness temperature' of the stars. For both stars it is about 2×10^7 deg K at 81·5 Mc/s and 10^6 deg K at 210 Mc/s.

ACKNOWLEDGMENTS

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APPENDIX A

THE EFFECT OF RADIATION FROM OTHER RADIO STARS

Determinations of the positions of radio stars by interferometric means are liable to errors due to the presence of other radio stars in the region of sky over which the aerials are receptive (Smith, in course of publication). A similar effect can occur in the measurement of angular diameter. Interferometric measurements may be regarded as giving the Fourier analysis of the distribution of intensity $I(\theta)$ across the sky, the amplitude of a record obtained at a single aerial spacing d being determined by the integral

$$\int I(\theta) \exp j \left\{ \frac{2\pi d}{\lambda} \theta \right\} d\theta$$

where the integral is taken over the region of sky over which the aerial is receptive. θ is here assumed to be small enough for sin θ to be replaced by θ. The measurement of diameter may be regarded as a measurement of the reduction of the value of this integral as d/λ is increased, and it has been shown that the actual distribution of intensity across a source of radiation such as the sun may be measured by observing the variation of the record amplitude as d is varied (Stanier 1950).

In measurements of the angular diameter of radio stars the minimum available value of λ/d may be large compared with the angular diameter of the star; γ will therefore only differ slightly from unity. At the same time, there may be an appreciable contribution to the record amplitude from the rest of the sky in the region observed, since there will be a random distribution of weaker radio stars and possibly some fine structure in the background intensity.

The amplitude of the signal to be expected from the weak stars has been calculated by Smith who shows that the ratio of the most probable record amplitude R_T from the 'interfering' stars to that (R_S) from the 'wanted' star is given by $R_T/R_S=(3p_S)^{1/2}$ where p_S is the probability of finding a star giving a record greater than or equal to R_S in the region of sky over which the aerial is receptive; p_S may be derived by extrapolating the observed distribution in intensity of the known radio stars (Ryle 1950).

The record obtained from a radio star may therefore have an amplitude differing from the true amplitude by a factor of up to $1 \pm R_T/R_S$; the amplitude may be increased or decreased according to the phase relation of the wanted and unwanted records. The value of R_T/R_S may often be so great that it considerably exceeds the reading accuracy, and it will therefore provide the most serious limitation to the measurement of angular diameter. For example, observations of the intense radio star in Taurus (05.01) with an interferometer comprising two aerials with gain 100 will be limited by a value of $R_T/R_S \simeq 0{\cdot}2$. (In this figure allowance has been made for the fact that some of the 'interfering' radio stars will have appreciably different declinations and will give records with different periodicities. This factor will make it possible to eliminate certain interfering stars.)

The resultant amplitude R_T is a function of the aerial spacing d. If the aperture of the individual aerials in the direction of the interferometer axis is a, R_T contains the Fourier components of the distribution corresponding to all aerial separations from $d-a$ to $d+a$. Since $I(\theta)$ is assumed to be a random function of θ it follows that if d is changed by an amount $2a$, the new value of R_T will bear a completely random relation in phase and amplitude to the previous one. If n separate values of d differing by more than $2a$ are used, the error due to the interfering stars may therefore be reduced by $1/n^{1/2}$.

APPENDIX B

§B.1. THE FREQUENCY CHARACTERISTICS OF AERIALS AND PREAMPLIFIERS

The receivers used for detecting the small signals from radio stars often use a large frequency bandwidth. The characteristics of the aerials and preamplifiers may vary over this band of frequencies, and this will affect the amplitude of the record obtained with any given pair of aerials. If the frequency responses of all the aerials and preamplifiers remain unchanged throughout all the observations, this is of no consequence. It has been found, however, that the preamplifiers are liable to change slowly, particularly in their frequency characteristics.

Suppose that associated with each of the three aerials of the interferometer there are preamplifiers, and that the preamplifiers in the fixed aerials are tuned to slightly different frequencies. Then the ratio of record amplitudes will depend critically on the tuning of the preamplifier in the common aerial. If in the course of the experiment the centre frequency of the preamplifier drifted from that of one fixed aerial to that of the other, a change in the ratio of the record amplitudes will be observed, which might be interpreted as a progressive change in Γ as the aerial is moved. It would of course be possible to check this by returning to a fixed position (such as the midway position) between each measurement, but it is simpler to make two separate determinations at each position, interchanging the preamplifiers between the measurements. The geometric mean of the two ratios may then be used, giving the correct ratio at each position.

§ B.2. THE EFFECT OF MISMATCH IN THE AERIAL TRANSMISSION LINES

In the theory of the phase-switching interferometer it has been assumed that the noise power in the receiver input included the total available power from the transmission lines connected to each aerial and its preamplifier. This will not be so if the transmission lines are mismatched to the receiver input, and furthermore if there are different amounts of mismatch in the transmission lines to the two fixed aerials A_1 and A_2, the measured values of Γ may be affected. Errors could then be introduced if the tuning of the main receiver changed during the experiment, since the mismatch in each long transmission line will be markedly dependent on frequency.

If the transmission lines from the preamplifiers at the aerials A_1 and A_2 to the receiver are of the same length and characteristic impedance, the error in Γ may at any time be reversed by interchanging the preamplifiers. In the experiments described above, the two transmission lines to aerials A_1 and A_2 were equal in length within a small fraction of a wavelength, and this effect could therefore be eliminated in this way.

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Tropospheric Wave Propagation in a Duct of Non-Uniform Height

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ABSTRACT. This paper describes an approximate method for extending the simple mode theory of tropospheric refraction (Booker and Walkinshaw 1947) to cover situations where the refractive index profile varies with range. The transmission path is divided into a number of zones, in each of which the variation in the refractive index profile is small. In each zone the profile is then taken to be independent of range, and given by the mean profile throughout the zone. The duct-width d and the lapse-rate of modified refractive index at infinite height κ_∞ are found for each zone and are used, together with published propagation curves, to evaluate the distribution of radio field strength in the region beyond the horizon of a transmitter. Although the method is a general one, the final formula refers specifically to situations in which the field can be adequately described by the first mode alone, and where the mean profile of refractive index in each zone may be represented analytically by a one-half power law curve.

An example of the application of the method to a practical situation is given. This shows good agreement between experiment and theory, and demonstrates the significant effect of changes in the refractive index profile on the level of field strength. The limitations of the method are pointed out, and its extension to cover other types of refractive index profile and higher order modes is outlined.

B
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§1. INTRODUCTION

A CHARACTERISTIC feature of the radio-meteorological situations observed over the sea in the neighbourhood of a coastline in many parts of the world is the change in refractive index profile with distance offshore. Allowance is not, however, made for this in the simple mode theory of tropospheric refraction (Booker and Walkinshaw 1947), which assumes that the profile of refractive index remains the same over the entire propagation path. Field strength distributions calculated on this basis do not agree satisfactorily with measurements. It becomes important, therefore, to extend the simple theory to include a profile which varies with distance.

In order to allow for such a profile we consider the transmission path as divided into a number of zones, in each of which the refractive index profile does not vary markedly with range. We then take the profile in each zone to be independent of range and given by the average profile throughout the zone. The actual refractive index distribution is thus reproduced with an accuracy which increases as the zone-lengths are decreased. The problem then reduces to deriving an expression for the field strength in terms of the refractive index profile in each zone and the range of each intervening boundary from the transmitter.

In a rigorous analysis it would be necessary to take account of reflections at zone boundaries. In practice, however, the fractional changes in refractive index are small, and for normal incidence the reflected energy may be regarded as negligible in comparison with the transmitted energy. In our analysis we shall therefore ignore the effect of reflections.

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§ 2. GENERAL EXPRESSION FOR THE FIELD STRENGTH

In the mode theory of tropospheric refraction Booker and Walkinshaw (1947) derive an expression for the horizontal component of electric field strength E, at range r from the transmitter, and height h, for an atmosphere that is horizontally stratified in refractive index. The field E is expressed as the sum of contributions from an infinite set of propagation modes, each of which may be described as the product of three terms characteristic of the refractive index profile:

$$E(r, h) = \Sigma_n a_n f_n(r) g_n(h). \qquad \ldots\ldots(1)$$

The term a_n can be thought of as the excitation of the nth mode, while $f_n(r)$ and $g_n(h)$ are the range and height dependent terms respectively.

Let the transmission path be divided into a number of zones, in each of which the refractive index profile is independent of range. Let the transmitter be situated in the first zone. The first object of this analysis is to derive a general expression for the field strength in any zone.

Let us consider the $(s+1)$th zone, which is separated from the previous (sth) zone by the sth boundary at range $r = r^{(s)}$. The field $E^{(s+1)}$ in the $(s+1)$th zone may again be expressed as the sum of mode contributions. The range and height dependent terms $f_n^{(s+1)}(r)$ and $g_n^{(s+1)}(h)$ may be made the same as in (1), and characteristic of the refractive index profile in the $(s+1)$th zone only. The excitation term must then depend on the profile along the entire transmission path. Let this term be denoted by $p_n^{(s+1)}$. Thus,

$$E^{(s+1)}(r, h) = \Sigma_n \, p_n^{(s+1)} f_n^{(s+1)}(r) g_n^{(s+1)}(h). \qquad \ldots\ldots(2)$$

In a similar notation, the field strength in the sth zone is given by

$$E^{(s)}(r, h) = \Sigma_m \, p_m^{(s)} f_m^{(s)}(r) g_m^{(s)}(h). \qquad \ldots\ldots(3)$$

If we write

$$g_m^{(s)}(h) = \Sigma_n \, t_{mn}^{(s)} g_n^{(s+1)}(h), \qquad \ldots\ldots(4)$$

then (3) becomes

$$E^{(s)}(r, h) = \Sigma_m \, \Sigma_n \, p_m^{(s)} f_m^{(s)}(r) t_{mn}^{(s)} g_n^{(s+1)}(h). \qquad \ldots\ldots(5)$$

We ignore reflections and use (2) and (5) to match the tangential electric field strengths on either side of the boundary at range $r = r^{(s)}$. Thus

$$p_n^{(s+1)} f_n^{(s+1)}(r^{(s)}) = \Sigma_m \, t_{mn}^{(s)} p_m^{(s)} f_m^{(s)}(r^{(s)}), \qquad \ldots\ldots(6)$$

where the transmission coefficient $t_{mn}^{(s)}$ is given, from (4) and the orthogonality of the height functions, by

$$t_{mn}^{(s)} = \frac{\int_0^\infty g_m^{(s)}(h) g_n^{(s+1)}(h) \, dh}{\int_0^\infty [g_n^{(s+1)}(h)]^2 \, dh}. \qquad \ldots\ldots(7)$$

Equation (6) is a reduction formula relating $p_n^{(s+1)}$ to the corresponding terms $p_m^{(s)}$ in the previous zone. In the first zone, containing the transmitter, $p_m^{(1)}$ is simply equal to $a_m^{(1)}$, as given in (1). Hence the field strength in any zone may be expressed in terms of functions given by the simple mode theory by means of (2), (6) and (7).

§ 3. SIMPLIFIED TREATMENT

Although an accurate solution involving a large number of small zones is theoretically possible, it is apparent from (6) that the problem becomes rather complex if more than two zones are involved. This complexity results from the interchange of energy between all the propagation modes at each zone boundary. It is, however, possible to bring in certain simplifications and at the same time retain a high degree of accuracy.

(i) *Assumptions Involved*

Consider a situation where the overall variation in refractive index profile has been lumped together into one discontinuity at a certain range, thus dividing the transmission path into two zones. If this discontinuity is small enough for us to assume that

$$\int_0^\infty [g_m^{(1)}(h)-g_m^{(2)}(h)]^2\,dh \ll \int_0^\infty [g_m^{(2)}(h)]^2\,dh, \qquad \ldots\ldots(8)$$

then it may be shown (see Appendix) that (7) yields the following approximate formula for the transmission coefficient from the *m*th mode in the first zone to the same mode in the second zone:

$$t_{mm}=\left\{\int_0^\infty [g_m^{(1)}(h)]^2\,dh \Big/ \int_0^\infty [g_m^{(2)}(h)]^2\,dh\right\}^{1/2}. \qquad \ldots\ldots(9)$$

The implications of (9) may be seen if we consider the energy flow arising from the *m*th mode in the first zone. From (4), we have

$$\int_0^\infty [g_m^{(1)}(h)]^2\,dh=\Sigma_n\, t_{mn}{}^2 \int_0^\infty [g_n^{(2)}(h)]^2\,dh,$$

and from (9) it follows that

$$\sum_{n\neq m} t_{mn}{}^2 \int_0^\infty [g_n^{(2)}(h)]^2\,dh=0. \qquad \ldots\ldots(10)$$

This means that the total energy exchange between different order modes at the zone boundary is negligible.

If the above is valid to a certain degree of accuracy for a given discontinuity in refractive index profile, it may be shown that it is valid to at least the same degree of accuracy for a series of smaller discontinuities representing the same overall change. We may therefore conclude that if the overall change in refractive index profile is such as to satisfy condition (8), then the energy in any transmission mode remains substantially in that mode throughout. This greatly simplifies the problem, since it means that each mode contained in the expression for the field strength in (2) may be treated separately. Consequently we can make the zone-lengths as small as we please, and thereby approximate very closely to the actual refractive index distribution, without introducing any complications. This simplification will be used in the next part of the analysis.

(ii) *Modified Expression for the Field Strength*

Since the modes are to be regarded as separable throughout, we can simplify the notation by omitting the sigmas and suffixes from the expressions for the field strength. Thus (1) and (2) may be written in the following shorthand form:

$$E(r,h)=af(r)g(h) \qquad \ldots\ldots(11)$$

for a refractive index profile independent of range, and

$$E^{(s+1)}(r,h)=p^{(s+1)}f^{(s+1)}(r)g^{(s+1)}(h) \qquad \ldots\ldots(12)$$

for a profile varying with range. Owing to the negligible interchange of energy between different modes, (6) reduces to

$$p^{(s+1)}f^{(s+1)}(r^{(s)})=t^{(s)}p^{(s)}f^{(s)}(r^{(s)}) \qquad \ldots\ldots(13)$$

which can readily be used as a reduction formula. Thus, if we write

$$k^{(l)} = f^{(l)}(r^{(l)}) / f^{(l+1)}(r^{(l)}), \qquad \ldots\ldots(14)$$

then

$$p^{(s+1)} = t^{(s)}k^{(s)}p^{(s)} = \left[\prod_{l=1}^{s} t^{(l)}k^{(l)}\right] a^{(1)}. \qquad \ldots\ldots(15)$$

Using (9), (12) and (15), we obtain the final expression for the field strength:

$$E^{(s+1)}(r, h) = G^{(s+1)}\left[\prod_{l=1}^{s} k^{(l)}\right] a^{(1)} f^{(s+1)}(r) g^{(s+1)}(h), \qquad \ldots\ldots(16)$$

where

$$G^{(s+1)} = \left\{\int_0^\infty [g^{(1)}(h)]^2\, dh \Big/ \int_0^\infty [g^{(s+1)}(h)]^2\, dh\right\}. \qquad \ldots\ldots(17)$$

The field strength has now been expressed as a function of the refractive index profile along the entire transmission path. By means of published propagation curves we can reduce the expression to a form which can readily be applied to practical situations.

§4. FORMULA IN TERMS OF A POWER-LAW REFRACTIVE INDEX PROFILE

All the terms in (17) are expressible in terms of functions already evaluated for a power-law refractive index profile of the form

$$\mu = 1 - \kappa_\infty d\,[h/d - (1/m)(h/d)^m], \qquad \ldots\ldots(18)$$

where μ is modified refractive index, as defined by Booker and Walkinshaw, κ_∞ is the lapse-rate of μ at infinite height, d is the duct-width, in the same units as h, and m is the profile index $(0 < m < 1)$.

The integrated square of the height function $g(h)$ in (17) may be expressed in terms of a tabulated function F as follows:

$$\int_0^\infty [g(h)]^2\, dh = d/2F. \qquad \ldots\ldots(19)$$

The expression for the field strength now reduces to

$$E^{(s+1)}(r, h) = \left[\frac{d^{(1)}}{d^{(s+1)}}\frac{F^{(s+1)}}{F^{(1)}}\right]^{1/2}\left[\prod_{l=1}^{s} k^{(l)}\right] a^{(1)} f^{(s+1)}(r) g^{(s+1)}(h). \qquad \ldots\ldots(20)$$

Curves giving the functions F, a, $f(r)$ and $g(h)$ have been published for the first mode and for both one-half and one-fifth power-law profiles ($m = \frac{1}{2}$ and $\frac{1}{5}$) by Booker and Walkinshaw (1947). Since the first mode is the one propagated with the least attenuation the field strength may often be adequately described in terms of this mode alone at ranges well beyond the horizon of the transmitter. These curves may therefore be used to calculate the field strength in many practical situations.

§5. PRACTICAL APPLICATION WITH FIRST MODE AND ONE-HALF POWER LAW PROFILE

(i) *General Formula*

In the decibel notation of Booker and Walkinshaw (1947) the terms F, a, $f(r)$ and $g(h)$ in (20) are expressed, for a one-half power law profile, as follows:

$$\left.\begin{aligned} 20\log_{10}F &= D_{\mathrm{E}} & \qquad 20\log_{10}f(r) &= D_{\mathrm{TR}}(r) \\ 20\log_{10}a &= D_0 + D_{\mathrm{E}} + D_{\mathrm{T}} & \qquad 20\log_{10}g(h) &= D_{\mathrm{R}} \end{aligned}\right\} \qquad \ldots\ldots(21)$$

From (20) and (21) the expression for the field strength in decibel form is

$$\begin{aligned}20 \log_{10} E(r, h) = {} & 10 \log_{10} d^{(1)} - 10 \log_{10} d^{(s+1)} + \tfrac{1}{2} D_{\mathrm{E}}^{(1)} + \tfrac{1}{2} D_{\mathrm{E}}^{(s+1)} \\ & + \sum_{l=1}^{s} [D_{\mathrm{TR}}^{(l)}(r^{(l)}) - D_{\mathrm{TR}}^{(l+1)}(r^{(l)})] + D_0^{(1)} + D_{\mathrm{T}}^{(1)} \\ & + D_{\mathrm{TR}}^{(s+1)}(r) + D_{\mathrm{R}}^{(s+1)}. \qquad \ldots\ldots(22)\end{aligned}$$

In some practical situations the duct width may be zero over part of the transmission path, and consequently some of the terms in (22) may become infinite. We may overcome this difficulty by using the published formulae of Booker and Walkinshaw (1947) for these terms as functions of the duct width. We may then group them together and evaluate the limit of their sum as the duct width tends to zero.

(ii) *Limitations*

The expression (22) for the field strength follows from the use of the approximate formula (9) for the transmission coefficient between zones. This coefficient has been calculated from the more accurate formula (7) for the first mode as a function of the variation in refractive index profile. A comparison with similar calculations from (9) enables us to estimate the extent of the error introduced when (9) is used instead of (7). The result is shown in fig. 1, where parametric curves indicate the magnitude of the overall variation in refractive index profile permissible before the error becomes as large as 1 or 2 db. The parameter L which is used is a function of the refractive index profile and the wavelength λ. It is tabulated by Booker and Walkinshaw (1947) and defined as follows:

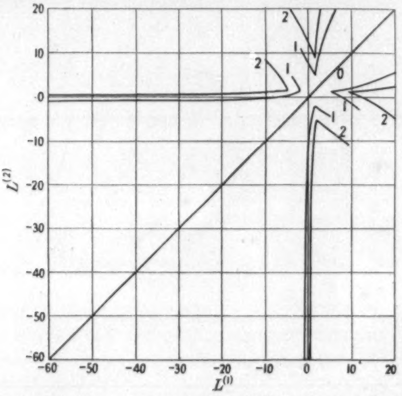

Fig. 1. Limitation diagram: parameter is upper limit of magnitude of error in db.

$$L = -20 \log_{10}[-3(n-\tfrac{1}{4})\lambda\kappa_\infty^{-1/2}(2d)^{-3/2}]. \qquad \ldots\ldots(23)$$

The ordinates $L^{(1)}$ and $L^{(2)}$ indicate the extreme values of L to be considered in any practical situation where the refractive index profile varies along the transmission path. For example, an overall variation in L from -20 to -1 would give an error of less than 1 db.

It is apparent from fig. 1 that if (22) is to be used, then large variations in L are permissible if the mode in question is either wholly trapped or wholly leaky along the entire transmission path. This is what we should expect from a more qualitative consideration of the problem, for the form of the height function $g_m(h)$ in condition (8), is not critically dependent on the refractive index profile except in the region of transition from trapped to leaky modes, where the magnitude of L is small.

§ 6. COMPARISON WITH EXPERIMENT

Figures 2 and 3 show an application of the theory to a set of experimental results. These results were obtained during an investigation in New Zealand into the formation and effects of radio ducts on u.h.f. radio wave propagation (Milnes and Unwin 1950), a report on which has recently been published by the New Zealand Government (1951).

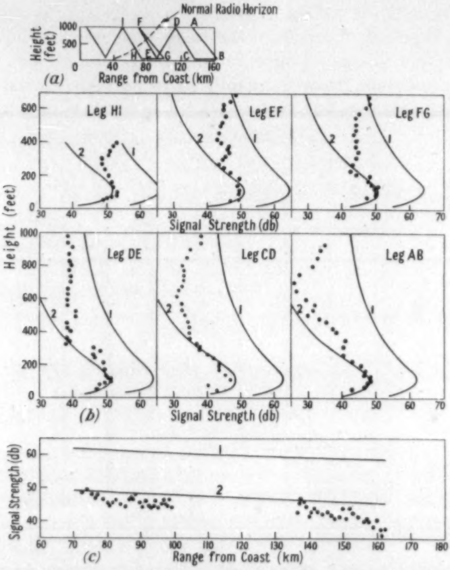

Fig. 2. Theoretical field strength in radio duct—comparison with observed values. (*a*) Elevation view of path covered by aircraft. (*b*) Field strength observations on slant legs of aircraft flight. (*c*) Field strength observations at 50 feet. Note: theoretical field strengths are for the first mode only and observed values reproduced are confined to those made below the normal radio horizon.

On 1st August 1947 a light north-west wind was blowing off the east coast of the South Island of New Zealand. An aircraft carrying u.h.f. transmitters and a psychrometer flew on a course perpendicular to the coast out to a maximum range of 160 km. An elevation view of the course is shown in fig. 2 (*a*). Observations of temperature and humidity, from which the refractive index could be calculated, were made from the aircraft several times a minute throughout the flight. At the same time measurements of field strength from the aircraft's transmitters were made at the coast. In addition to the course shown in fig. 2 (*a*) the aircraft made a horizontal run well above the radio horizon in the region covered by the interference lobe pattern, from which a free-space level for each radio channel could be established. A second aircraft made further meteorological observations in the region up to 100 km offshore.

The field strength observations made below 1 000 feet at a wavelength of 58 cm are plotted in figs. 2 (*b*) and 2 (*c*), the former giving the results obtained during the slant legs of the flight, and the latter those made while the aircraft was flying at approximately 50 feet from the surface. The decibel scale has been arbitrarily adjusted so that the free space level, which was established with an accuracy of ±2 db, is 50 db at 100 km range. Only those observations made below the normal radio horizon have actually been plotted at the smaller ranges.

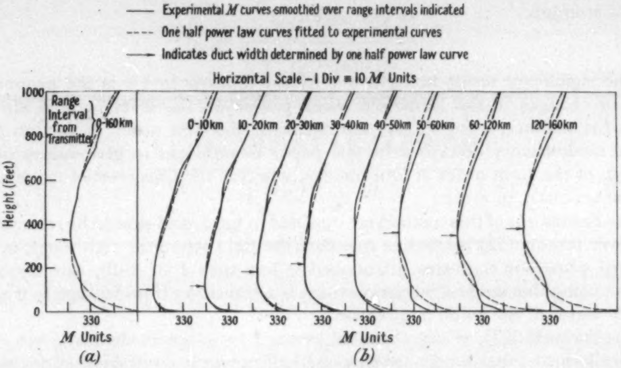

Fig. 3. *M*-curves. (*a*) Average *M*-curve 0–160 km. (*b*) Average *M*-curves in zones.

The meteorological observations showed that the duct increased rapidly in width from about 100 feet near the coast to about 350 feet at 55 km. Thereafter it remained practically constant out to about 120 km, increasing gradually again to over 400 feet at 160 km.

The experimental refractive index values were first smoothed over the whole transmission path in height intervals of 50 to 100 feet up to 1 000 feet, the resulting average *M* curve* being shown as a full line in fig. 3 (*a*). A one-half power law curve fitted by a 'least squares' method to the smoothed experimental values is shown by the dashed curve on the same diagram. The duct width is 330 feet. The theoretical field strength for the first mode at 58 cm wavelength was then calculated along the path covered by the aircraft, by means of the simple mode theory of Booker and Walkinshaw (1947). The mode is just trapped for the above refractive index profile. The results calculated from (11) and (21) are shown in the curves 1 of figs. 2 (*b*) and 2 (*c*). In all cases the predicted field strength is some 12 to 15 db above the experimental value.

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* M is defined as $(\mu-1)10^6$, where μ is the actual refractive index suitably modified so that the earth may be regarded as flat.

The transmission path was then divided into a number of zones, in each of which the change in refractive index profile was relatively small. The zones are necessarily shortest where the changes in refractive index profile are most rapid. Thus the zones are each 10 km long in the first 60 km from the coast, but much longer at greater ranges. The average M curves in each zone are shown by the full line curves of fig. 3 (*b*), while the one-half power law curves fitted to these profiles are shown by the dashed curves. The first mode passes from a leaky to a trapped state at about 50 km from the coast, and we should therefore expect errors ranging up to more than 2 db, owing to the limitations of the method described in § 5 (ii). The results calculated from (22) are shown in curves 2 of figs. 2 (*b*) and 2 (*c*), and show a striking agreement between the theoretical and experimental values for the field strength in the first few hundred feet from the surface. Above this interval higher order modes begin to influence the level of field strength.

§ 7. REMARKS

The significant result brought out by the analysis in § 6 is the pronounced effect of changes in the refractive index profile on the level of field strength. When the conditions for its use are satisfied, the first order correction to the simple mode theory described in this paper is sufficient to give values of field strength of the right order at long ranges, whereas the uncorrected mode theory may be seriously in error.

The limitations of this method are outlined in fig. 1, and should be remembered whenever fomula (22) is used to calculate the field strength. Although in most practical situations the error introduced is less than 1 or 2 db, care should be taken in using this method whenever there is a transition from leakage to trapping (or vice versa) of the mode in question.

The formula (22), where the field strength is given in decibel form, refers specifically to the first mode and to a one-half power law refractive index profile. Published propagation curves, however, make it equally possible to derive a similar formula from (20) for a one-fifth power law profile. Moreover, the expression (16) is perfectly general, and could be applied to any mode and even to an arbitrary profile, once curves giving the functions a, $f(r)$ and $g(h)$ were available. The main work in preparing these curves would be the calculation of the eigenvalues, for which a variational method has been described by Macfarlane (1947, 1948).

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APPENDIX

APPROXIMATE FORMULA FOR THE TRANSMISSION COEFFICIENT

The transmission coefficient from the *m*th mode in the first zone to the same mode in the second zone is obtained from (7). Thus,

$$t_{mm}=\frac{\int_0^\infty g_m^{(1)}(h)g_m^{(2)}(h)\,dh}{\int_0^\infty [g_m^{(2)}(h)]^2\,dh}. \qquad \ldots\ldots(24)$$

If condition (8) is satisfied, then (24) may be written as follows:

$$t_{mm}=\frac{\frac{1}{2}\int_0^\infty [g_m^{(1)}(h)]^2\,dh+\frac{1}{2}\int_0^\infty [g_m^{(2)}(h)]^2\,dh}{\int_0^\infty [g_m^{(2)}(h)]^2\,dh}$$

$$=1+\frac{\frac{1}{2}\int_0^\infty [g_m^{(1)}(h)]^2\,dh-\frac{1}{2}\int [g_m^{(2)}(h)]^2\,dh}{\int_0^\infty [g_m^{(2)}(h)]^2\,dh}. \qquad \ldots\ldots(25)$$

Within the given restrictions we may regard the right-hand side of (25) as the binomial expansion of the square root of a quantity very close to unity. Thus,

$$t_{mm}=\left\{1+\frac{\int_0^\infty [g_m^{(1)}(h)]^2\,dh-\int_0^\infty [g_m^{(2)}(h)]^2\,dh}{\int_0^\infty [g_m^{(2)}(h)]^2\,dh}\right\}^{1/2}$$

$$=\left\{\int_0^\infty [g_m^{(1)}(h)]^2\,dh\Big/\int_0^\infty [g_m^{(2)}(h)]^2\,dh\right\}^{1/2}, \qquad \ldots\ldots(26)$$

which is the result given in (9).

LETTERS TO THE EDITOR

Temperature Variation of the Rectification Characteristics at Selenium Contacts

One of the parameters determining the characteristics of rectifying junctions is the diffusion potential V_D, defined as the height of the potential barrier in the semiconductor in the absence of applied field. V_D enters into the theoretical equations relating barrier voltage U and current density j and it can, in principle, be evaluated by substituting experimental values into these equations. The purpose of the present experiments was to test the validity of theoretical voltage–current characteristics and to evaluate V_D from them at different temperatures. The measurements were carried out on contacts between selenium of high purity and gold, applied by vacuum evaporation. The reverse characteristics of these junctions were analysed in accordance with the expressions given by Mott (1939) and Landsberg (1951). Both sets of equations are based on the diffusion theory and embody corrections for the effect of the image force. They assume that only one type of charge carrier is present. The expressions under discussion are :

Mott : $$j/U=(1/\rho_\infty)(1+V_D/U)\exp[B(V_D+U)^{1/2}-BV_D^{1/2}],$$

where $$1/\rho_\infty=(\sigma/\lambda)\exp(-\boldsymbol{e}V_D/\boldsymbol{k}T) \quad \text{and} \quad B=\boldsymbol{e}^{3/2}/\boldsymbol{k}T(\kappa\lambda)^{1/2};$$

Landsberg : $$j/j_0=\{1-\exp(-\boldsymbol{e}U/\boldsymbol{k}T)\}y^{3/2}\exp(y),$$

where $$j_0=(2/\pi)^{1/2}Nb\kappa(\boldsymbol{k}T/\boldsymbol{e})^2\exp(-\boldsymbol{e}V_D/\boldsymbol{k}T);$$

$$\left.\begin{array}{lcl} y=\{(V_D+U)/V_0\}^{1/2} & \text{or} & \{(V_D+U)/V_0\}^{1/4} \\ V_0=(\kappa\lambda/\boldsymbol{e})(\boldsymbol{k}T/\boldsymbol{e})^2 & \text{or} & (\kappa^3/8\pi N\boldsymbol{e}^3)(\boldsymbol{k}T/\boldsymbol{e})^4 \end{array}\right\} \text{for a Mott or Schottky barrier respectively.}$$

where N is the concentration of impurity centres, λ the barrier thickness, b the carrier mobility, σ the conductivity of the semiconductor and κ its dielectric constant. The remaining terms have their usual meaning. The applied voltage is regarded as positive in the reverse direction. The quantities (ρ_∞, V_D, B) and (V_0, j_0, V_D) were evaluated (to about ±10%) by fitting the equations to the experimental curves at three selected and representative points.

All equations tested gave good fits at room temperature and above, but not always at temperatures below 0° c. Figures 1 and 2 show typical results. The constants evaluated for one contact were

	Mott's equation			Landsberg's equations					
	V_D (v)	B ($v^{-1/2}$)	ρ_∞ (Ω cm²)	V_D (v)	V_0 (v)	j_0 (A/cm²)	V_D (v)	V_0 (v)	j_0 (A/cm²)
60° c	1·2	1·84	2860	0·47	0·476	$1{\cdot}2\times10^{-4}$	0·53	$2{\cdot}0\times10^{-3}$	$7{\cdot}06\times10^{-7}$
20° c	0·26	2·09	4620	0·096	0·293	$5{\cdot}3\times10^{-5}$	0·105	$1{\cdot}53\times10^{-3}$	$4{\cdot}74\times10^{-7}$
−42° c	0·012	1·85	2570	Very low	—	—	Very low	—	—
				Mott barrier			Schottky barrier		

V_D as determined by this method is seen to decrease with decreasing temperature. At the lowest temperature the results obtained are without meaning, since the equations assume that $V_D>\boldsymbol{k}T/\boldsymbol{e}$. The present values of V_0 and j_0 are verv similar to those given by Landsberg (1951) and imply $N=8{\cdot}5\times10^{17}$ per cm³, $b=1{\cdot}5\times10^{-2}$ cm²/volt sec, and (for the Mott barrier) $\lambda=7{\cdot}5\times10^{-6}$ cm. The curves by themselves do not show which type of barrier model is the better approximation, although from capacitance measurements it must be inferred that the barrier is one of variable thickness, i.e. of the Schottky or composite type.

The present results show that the equations quoted are a satisfactory representation of the reverse current–voltage relations at a given temperature. The interpretation of the 'effective diffusion potentials' here evaluated is, however, still in need of further clarification, particularly since V_D is found to be temperature dependent in the sense opposite to that expected from theoretical considerations (Landsberg 1952). The anomalous behaviour of

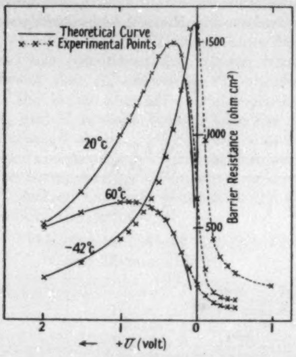

Fig. 1. The fit obtainable between an experimental reverse characteristic and Mott's equation.

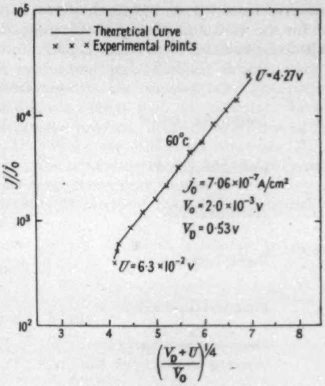

Fig. 2. The fit obtainable between an experimental reverse characteristic and Landsberg's equation for a Schottky barrier.

the effective V_D is probably due to oversimplification of the conditions which prevail at normal rectifying contacts, in that other factors besides the image force may have to be considered.

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17th September 1952.

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Thermo- and Galvano-Magnetic Coefficients for Semiconductors

Formulae derived using classical statistics for the thermo- and galvano-magnetic coefficients have been given by Sommerfeld and Frank (1931) and by Bronstein (1932). These formulae, together with some other relevant formulae are summarized in the table.

Sommerfeld and Frank showed their formulae gave the correct order of magnitude when applied to metals (although better agreement was obtained with formulae derived from quantum statistics). Wright (1951) has recently attempted to discuss the properties of semiconductors using the formulae given by Sommerfeld and Frank. These formulae do not lead to the correct conclusions when applied to semiconductors because they are derived under the assumption that the only significant contribution to the thermal conductivity is the electronic one. Although this assumption may be applied to metals, in a semiconductor the lattice contribution to the thermal conductivity will normally be many times larger than the electronic. Bronstein has taken this into account in deriving his formulae. Comparison of his formulae with those of Sommerfeld and Frank shows that the same results are obtained for the Hall and Nernst coefficients but Bronstein's formulae for the Ettingshausen and Righi–Leduc coefficient can be written $P=P_0K_0/K$, $S=S_0K_0/K$ where K is the thermal conductivity, K_0 is the electronic contribution to it, and P_0 and S_0 are the formulae for the Ettingshausen and Righi–Leduc effects given by

Sommerfeld and Frank. Thus these results predict that while the classical formulae should hold for the Hall and Nernst coefficients, the Ettingshausen and Righi–Leduc coefficients will both be reduced by a factor K_0/K.

Coefficient	Sommerfeld & Frank	Bronstein
Isothermal Hall R	$\frac{3\pi}{8}\frac{1}{n\mathbf{e}}$	$\frac{3\pi}{8}\frac{1}{n\mathbf{e}}$
Ettingshausen P	$\frac{3\pi}{32}\frac{1}{n\mathbf{k}}$	$\frac{3\pi}{16}\frac{\mathbf{k}T}{n\mathbf{e}^2}\frac{\sigma}{K}$
Isothermal Nernst Q	$\frac{\pi}{4}\frac{l\mathbf{k}}{(2\pi\mathbf{mk}T)^{1/2}}$	$\frac{3\pi}{16}\frac{\mathbf{k}}{n\mathbf{e}^2}\sigma$
Righi–Leduc S	$\frac{7\pi}{16}\frac{l\mathbf{e}}{(2\pi\mathbf{mk}T)^{1/2}}$	$\frac{21\pi}{32}\frac{\mathbf{k}^2T}{n\mathbf{e}^3}\frac{\sigma^2}{K}$
Electrical conductivity σ	$\frac{4}{3}\frac{ln\mathbf{e}^2}{(2\pi\mathbf{mk}T)^{1/2}}$	
Electronic contribution to thermal conductivity K_0	$\frac{8}{3}\frac{ln\mathbf{k}^2T}{(2\pi\mathbf{mk}T)^{1/2}}$	

In these formulae T is the absolute temperature, n is the concentration and l the mean-free path of the conduction electrons. In Bronstein's formulae σ is the electrical conductivity and K the thermal conductivity.

There is a simple physical explanation for the fact that the Nernst effect obeys the classical formula but the Ettingshausen and Righi–Leduc effects do not. In the Nernst effect a longitudinal temperature difference is set up in the specimen. This produces an energy flow which can be resolved into an electronic and a lattice contribution. The electronic component will be that predicted by the classical calculations. The transverse voltage which appears when a magnetic field is applied depends only on the electronic component and is not affected by the presence of the lattice component so that the classical formulae will still hold. In both the other effects a transverse temperature difference appears in the specimen. This causes a transverse flow of energy which balances the transverse component of energy flow produced by the distortion of the longitudinal electronic flow by the magnetic field. If the temperature difference is ΔT, then this will produce an energy flow proportional to $K\Delta T$. If the thermal conductivity were entirely electronic then the energy flow would be proportional to $K_0\Delta T_0$ where ΔT_0 is the temperature difference predicted by the classical formulae. Since we assume that the longitudinal electronic current is not affected by the longitudinal lattice energy flow, then we must have $K\Delta T = K_0\Delta T_0$ and hence the classical formulae must be multiplied by the factor K_0/K to give the observed temperature difference.

The importance of the factor K_0/K can be illustrated first by considering germanium. Grieco and Montgomery (1952) have found $K = 0{\cdot}033$ watt cm^{-1} deg^{-1} at 25° C. For very impure germanium σ could be 100 ohm^{-1} cm^{-1} at this temperature. Using $K_0/\sigma = 2\mathbf{k}^2T/\mathbf{e}^2$ one obtains $K_0 = 4{\cdot}3\times10^{-4}$ watt cm^{-1} deg^{-1} and thus $K_0/K \sim 10^{-2}$. Thus in germanium the Ettingshausen and Righi–Leduc effects will be considerably smaller than predicted by the classical formulae. A second example is given by the following experiment made on a specimen of lead selenide. The Hall coefficient and the electrical and thermal conductivities were determined. From these results the three remaining coefficients could be calculated using both the classical and Bronstein's formulae and the results compared with the experimentally determined values. These numerical data are summarized as follows:

(*a*) Primary data: ($T = 300$° K), $R = 8{\cdot}0$ cm^3 coulomb^{-1}, $\sigma = 150$ ohm^{-1} cm^{-1}, $K = 4{\cdot}2\times10^{-2}$ watt cm^{-1} deg^{-1}.

(*b*) Comparison of theory and experiment (practical units)

Coefficient	Classical	Bronstein	Measured
Nernst (at $T = 358$° K)	$2{\cdot}7\times10^{-2}$		$2{\cdot}3\times10^{-2}$
Ettingshausen	$2{\cdot}3\times10^{4}$	$3{\cdot}7\times10^{2}$	$4{\cdot}4\times10^{2}$
Righi–Leduc	1050	17	25

It is seen that the measured values are of the order predicted by Bronstein's formulae but the classical formulae give values about 100 times too large.

Callen (1948, 1952) has recently discussed these effects from a thermodynamic standpoint and has considered the relation between the isothermal and adiabatic effects. He has shown that $R_a - R_i = P\,d\theta/dT$ where $d\theta/dT$ is the thermoelectric power. The presence of the lattice thermal conductivity in semiconductors means that the difference between R_a and R_i is much smaller than the classical formulae predict. For the specimen of PbSe discussed above $P\,d\theta/dT$ was about 1% of R. Thus the difference between R_a and R_i is comparable with the experimental error and the error introduced by the presence of the Ettingshausen effect is not as serious as has been sometimes supposed.

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26th September 1952.

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Intrinsic Conduction in PbS, PbSe, PbTe

In a recent Letter (Putley 1952) we summarized the results obtained from our studies of the electrical properties of single crystals of PbS, PbSe and PbTe. In that Letter we were unable to give a value for the gap between the full and conduction bands for PbSe, but we have now measured the Hall coefficient and conductivity of PbSe in the intrinsic range. These results will be described in more detail later, but the general behaviour is similar to that already reported for PbTe (Chasmar and Putley 1951) and PbS (Putley and Arthur 1951).

The gap between the bands for PbSe was calculated from the Hall effect data using the formula $R = AT^{-3/2}\exp(\epsilon/2\boldsymbol{k}T)$. These measurements also provided further evidence for the $T^{-5/2}$ mobility law. Using this mobility law leads to the formula for the intrinsic conductivity $\sigma = BT^{-1}\exp(-\epsilon/2\boldsymbol{k}T)$. The values for the gap obtained from these formulae are given in the table, together with the values found for PbS and PbTe. These values hold for $T > T_{\text{min}}$.

Gap between Full and Empty Bands (ev)

	Calc. from R	Calc. from σ	T_{min} (° K)
PbS	1·17	1·05	650
PbSe	0·50	0·45	750
PbTe	0·63	0·54	500

It is seen that the values deduced from the conductivity measurements are consistently smaller than those found from the Hall effect, but from both calculations the gap for PbSe is found to be smaller than that for PbTe.

These results show that in these compounds the values of the gaps between the bands do not come in the expected order but in the same order as the long-wave limits of photoconductivity (Gibson, Lawson and Moss 1951).

We wish to thank Mr. W. D. Lawson for supplying the single crystals, Mr. W. H. Mitchell for designing and constructing the crystal holders and Miss M. Stallard for assisting with the measurements. Acknowledgment is made to the Chief Scientist, Ministry of Supply, for permission to publish this letter.

Telecommunications Research Esatblishment, E. H. Putley.
Great Malvern, Worcs.
26th September 1952.

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Gibson, A. F., Lawson, W. D., and Moss, T. S., 1951, *Proc. Phys. Soc.* A, **64**, 1054.
Putley, E. H., 1952, *Proc. Phys. Soc.* B, **65**, 388.
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The Measurement of Drift Mobility in Semiconductors

While reproducing the experiments made at the Bell Telephone Laboratories (Haynes and Shockley 1949, 1951, Shockley *et al.* 1949) in order to determine the drift mobility of injected carriers in germanium, we have found it advantageous to make certain modifications to the methods used by these workers.

Haynes and Shockley (1951) have discussed the advantage of using a pulsed sweep field to reduce heating of the filament, but in combining this with a pulsed emitter current they have limited the transit time which may be measured. Furthermore, corrections must be made for the effect of the emitter current and conductivity modulation on the sweep field, and the transit time must be measured to the point of steepest slope on the arrival or departure waves which is difficult to locate on a cathode-ray tube display.

We have used a direct emitter current of about 0·2 mA and a pulsed sweep field of up to 50 v/cm and 140 μsec duration. During the relatively long time between sweep pulses, the emitter builds up about itself an equilibrium distribution of carriers falling off as exp $[-x/(D\tau)^{1/2}]$ where x is the distance from the emitter, τ the carrier lifetime, and the diffusion constant $D=\mu \boldsymbol{k}T/\boldsymbol{e}$.

When the sweep pulse arrives, this ‘ packet ’ of carriers is swept bodily down the filament where it is detected by the collector. As the emitter current is small the emitter cannot replace the holes as fast as they are swept down the filament and the resultant display of collector current consists of a single peak. The transit time, which is easily measured from the beginning of the sweep pulse to the top of the carrier arrival peak, can be made almost as long in duration as the sweep field pulse. No corrections need be made for conductivity modulation or alteration of the sweep field by the emitter current. A further advantage is that the end of the filament need not be treated to reduce end injection because the end injected carriers are always separated in space from those injected from the emitter.

The mobility is calculated in the usual way after a correction for non-uniformity of the field, as described by Haynes and Shockley (1951). Figure 1 shows the velocity of holes plotted against the field in n-type germanium. After the correction for non-uniformity of the field has been applied, the final result, namely 1700 cm²/volt sec, is in good agreement with the values obtained at the Bell Telephone Laboratories.

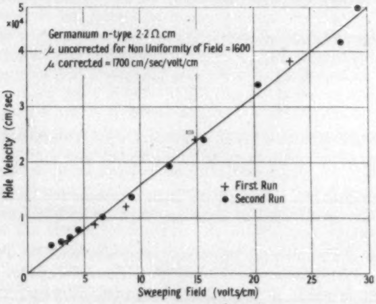


Fig. 1. Mobility determination by pulsed field, d.c. emitter method.

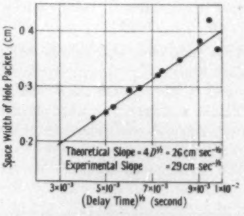


Fig. 2. Pulsed field d.c. emitter diffusion experiment.

It is a well-known feature of the theory of photodiodes that illumination within the main absorption band can produce minority carriers in the semiconductor. Clearly therefore the d.c. emitter in the above arrangement can be replaced by a light spot and the resultant output is unchanged. This has been confirmed by experiment on germanium. In this arrangement, therefore, the rather special conditions required for carrier injection from metal connections need not be achieved and the drift mobility can in principle be measured in any semiconductor showing rectification and photovoltaic effects. In practice however the efficiency of the collector may also be very low in some materials. In this case the ohmic pulse from the sweeping current overloads the output display before the desired signal can be detected. It has been found desirable to feed a cancelling pulse to the display to reduce this effect.

To confirm that the width of the carrier arrival pulse at the collector is that expected from diffusion, the pulse width is measured at the collector in units of time and this is converted to a space width using the measured velocity of the holes. The space width of the hole ' packet ' may be defined as the width at $1/e$ of the peak magnitude. Then at $t=0$ the space width is $2(D\tau)^{1/2}$. The space width after a finite delay time t involves a convolution and we are indebted to Mrs. A. M. Woodward of the Mathematics Division, Telecommunications Research Establishment, for the solution. In the simple case when $t \gtrsim \tau$ the space width of the packet should increase linearly with $t^{1/2}$, the factor of proportionality being $4D^{1/2}$. Using a mobility value of 1700, $4(D)^{1/2}$ is equal to 26 cm sec$^{-1/2}$. The experimental results, given in fig. 2, indicate a slope of 29 cm sec$^{-1/2}$. This discrepancy, which implies that the spreading out of the hole packet is slightly greater than that expected from diffusion alone, can probably be ascribed to either conductivity modulation, non-uniform sweeping field, or the effect of the small but continuous d.c. emitter current, or all three.

Acknowledgment is made to the Chief Scientist, Ministry of Supply, and the Controller of H.M. Stationery Office for permission to publish this letter.

Telecommunications Research Establishment,
Great Malvern, Worcs.
15th September 1952.

R. LAWRANCE.
A. F. GIBSON.

HAYNES, J. R., and SHOCKLEY, W., 1949, *Phys. Rev.*, **75,** 691; 1951, *Ibid.*, **81,** 835.
SHOCKLEY, W., HAYNES, J. R., and PEARSON, G. L., 1949, *Bell. Syst. Tech. J.*, **28,** 344.

Long Wavelength Infra-Red Photoconductivity of Silicon at Low Temperatures

During recent years we have made a number of measurements on semiconductors in order to determine whether it is possible to obtain appreciable photoconductive effects due to optical excitation of impurities. In experiments on silicon at low temperatures we have observed a photoconductive response for wavelengths in the range $2\,\mu$ to $14\,\mu$ which we believe can be attributed to this cause. The measurements were made with specimens of p-type material containing between 10^{16} and 10^{17} acceptors per cm^3 with an ionization energy of the order of 0·05 ev. Temperatures of the order of 20° K or below were required, since, at higher temperatures, the conductivity was too large for the effect to be observable.

The specimen was contained in a cryostat, cooled by liquid hydrogen, and radiation from a rock salt monochromator was directed on to it. The radiation was interrupted by a rotating sector disc. The dimensions of the specimen were of the order of a few millimetres and, at 20° K, the resistance was between 10^8 and 10^9 ohms. A potential of about 100 V was applied to the specimen through a feed resistor of 100 megohms and the specimen was connected to a tuned amplifier having a high input impedance and an effective bandwidth of about 1 c/s. Measurements have been made on single crystals and also on polycrystalline material.

For a given intensity of radiation, the photoconductive response increased with increasing wavelength between $2\,\mu$ and $14\,\mu$. This increase can be attributed to the increase in absorption in the specimen and the increase in the number of quanta for a given energy. At $2\,\mu$ only a small fraction of the incident radiation is absorbed in a thickness of the order of a millimetre, but the absorption, due to impurity ionization, increases rapidly as the wavelength approaches the ionization limit. The wavelength corresponding to the ionization energy of 0·05 ev is about $25\,\mu$ so that the long wavelength limit should lie somewhere in this region. At present our measurements have extended only as far as 14 μ. At 10 μ, with an interruption frequency of 800 c/s and the specimen at 20° K, a signal of the order of 100 microvolts was obtained for an incident intensity of 1 μW/cm². As there was no appreciable difference between the signals obtained with interruption frequencies of 20 and 800 c/s, the response time is less than 2×10^{-4} second. From the magnitude of the signal and the response time, the possibility that the response is due to a bolometer effect can be excluded.

At low frequencies the major contribution to the noise voltage across the specimen was due to fluctuations of the resistance of the specimen and the feed resistor. Since the power spectrum of this type of noise is inversely proportional to frequency, the signal to noise ratio was better at 800 c/s than at 20 c/s. Under conditions of optimum signal to noise ratio, the minimum intensity of 10 μ radiation which could be detected was of the order of 10^{-8} watt/cm².

A full account of this work will be given in a later publication.

The Clarendon Laboratory,
Oxford.
15th October 1952.

B. V. Rollin.
E. L. Simmons.

On a Method for Measurements of Optical Constants by Reflection

In a recent article, Avery (1952) has indicated a method of obtaining the optical constants of absorbing materials from reflectivity measurements. The ratio ρ^2 of the reflectivities for incident vibrations parallel and perpendicular to the plane of incidence is measured for at least two different angles of incidence, and the complex refractive index $N=n-ik$ is found by interpolation on curves drawn beforehand. As a complement to Avery's work we wish to point out that N can also be derived from the same measurements by solving a cubic equation without any need for graphs. To do this, if n_0 is the index of the first medium (generally air, $n_0=1$) we can use the following approximate formulae, which are however sufficiently exact in our case, to find a and b where

$$(a-ib)^2=N^2-n_0^2\sin^2\theta;\qquad a^2=n^2\left(1-\frac{n_0^2\sin^2\theta}{n^2+k^2}\right);\qquad b^2=k^2\left(1+\frac{n_0^2\sin^2\theta}{n^2+k^2}\right).$$

Using these and the expression for ρ^2 given by Avery, a simple algebraic calculation shows that n is a solution of the following cubic equation :

$$[(\Delta b)^2+a_1\Delta a\Delta b-b_1(\Delta a)^2]n^3+[-(c_1\Delta a+2\Delta d+a_1\Delta c)\Delta b+(2b_1\Delta c+d_1\Delta a-a_1\Delta d)\Delta a]n^2$$
$$+[(\Delta d+a_1\Delta c+c_1\Delta a)\Delta d+(c_1\Delta b-2d_1\Delta a-b_1\Delta c)\Delta c]n+(d_1\Delta c-c_1\Delta d)\Delta c=0,$$

where

$$a=2\frac{1+\rho^2}{1-\rho^2}n_0\sin\theta\tan\theta,\qquad c=n_0^2\tan^2\theta,$$
$$b=2n_0^2\sin^2\theta,\qquad d=\tfrac{1}{2}an_0^2\sin^2\theta=\tfrac{1}{4}ab.$$

Δ represents the difference (for each quantity) obtained for two different angles; and the subscript 1 the values of a, b, c and d for one of these angles of incidence. If n is known k is derived from the relation

$$k=\left(\frac{\Delta an^2+(\Delta b-\Delta c)n-\Delta d}{-\Delta an+\Delta c}n\right)^{1/2}.$$

The advantage of these formulae lies in the fact that it is no longer necessary to evaluate ρ^2 in terms of the three variables n, k and θ, for these calculations are relatively long, and further, there is no need for interpolation. It seems more logical to use the solution of an algebraic cubic equation rather than interpolation in a function of three variables.

On performing this calculation with $\rho^2=0{\cdot}24$ for $\theta=60°$ and $\rho^2=0{\cdot}08$ for $\theta=80°$ we find $n=3{\cdot}63$ and $k=1{\cdot}65$, while Avery, using three values of θ, found $n=3{\cdot}68\pm0{\cdot}05$, and $k=1{\cdot}58\pm0{\cdot}09$.

Institut d'Optique,
Paris.
17th October 1952.

F. Abelès.

Avery, D. G., 1952, *Proc. Phys. Soc.* B, **65**, 425.

[Dr. Avery informs us that, in addition to the data quoted earlier, graphs relating ρ^2 to n and k are now available for angles of incidence 30°, 60°, 73°, 80° and 83°. These graphs cover a range in n from 1·0 to 6·0, and in k from 0 to 10.—Editor.]

OBITUARY NOTICES

DONALD KENNETH ASHPOLE

DONALD K. ASHPOLE received his earlier education at Bishop's Stortford College and entered Imperial College in 1942. Two years later he obtained an honours degree in physics and after leaving college spent two years with British Fillers Ltd. He was then appointed assistant physicist at Courtauld's research laboratories at Bocking, but after a short time became senior physicist in charge and held this post up to his untimely death on 3rd March 1952, at the early age of 27 years.

Donald Ashpole's later scientific work dealt with dyeing and moisture relations in textile fibres. He had been invited by the Royal Society to read a paper on this subject and, but for his illness, this would have been delivered by him a few days before his death. In his spare time Ashpole was actively concerned in church work and had sat on the Free Church Federal Council ; he had also produced pageants and plays with much success. To his tutors at college Donald will be remembered for his pleasant and happy temperament and his sincere affection for those who had helped him forward in his scientific career. With his wife and two children we mourn his passing.

R. W. B. STEPHENS.

GRANVILLE HUGH BAILLIE

GRANVILLE HUGH BAILLIE was born in 1873 at Bath. He was educated at a private school (Winton House, Winchester) and entered the City and Guilds College in 1890 winning the Institute's Scholarship. He completed the course in Electrical Engineering in 1893 and was awarded the Diploma of Associate of the City and Guilds of London Institute, (A.C.G.I.).

He later attended a post-graduate course at the Hochschule at Zürich. It was there that he met the lady who became his wife.

His first important post was that of Managing Director of the Riviera Electricity Company Bordighera. He made himself responsible for the installation and maintenance of the complete electrical plant, a position requiring much energy and initiative in a young man of twenty-five.

On his return from Italy he entered into partnership with Mr. James Swinburne, F.R.S. (later Sir James Swinburne, Bart), as a consulting engineer, and they were later joined by Mr. Mervyn O'Gorman. About 1910, this firm having been dissolved, he formed with Mr A. Dobree the firm of Dobree and Baillie as consulting engineers and specializing in patent law.

Baillie's equable temper fitted him eminently for partnership. He and Dobree were both of them exceptional amateur mechanics, Baillie being also a keen exponent of ornamental turning.

Baillie was an excellent draughtsman and drew the majority of the drawings for the patent specifications himself. He often appeared as an expert witness and it was in that capacity that he was the first to use a cinematograph film as evidence in the High Court.

With the outbreak of the 1914–18 war the firm was dissolved and Baillie worked at the Ministry of Munitions on numerous problems connected with the dilution of labour. For this work he was awarded the O.B.E.

After the war Baillie returned to private practice as a consulting engineer. He had been connected with the British Tabulating Machine Company since 1910 and was appointed Joint Managing Director for the period of the war, 1939–45. He acted as Consulting Engineer to the Company until his death.

Early in life his appreciation for fine mechanisms attracted him to a close study of watches, incidental to which he gradually collected an excellent horological library.

In 1929 he produced his first book *Watches, their History, Decorations and Mechanisms.* This was a large and handsome book published by Methuen and Co., in their Connoisseur Library Series. It was followed in the same year, in the same series, by *Watchmakers and Clockmakers of the World.* This contained 25 000 names. In 1947 a second edition of the latter book was published by the N. A. G. Press containing 35 000 names and a great deal more information. The work involved in collecting and sorting out such a number of names was immense.

A few months before his death his *Clocks and Watches, a Horological Bibliography* was published by the N. A. G. Press. It is perhaps the most authoritative book on the subject, and won for Baillie the Gold Medal of the British Horological Institute.

Baillie rearranged and catalogued the collection of clocks and watches belonging to the Clockmakers Company at the Guildhall, and also the books in the library relating to horology. Whilst engaged on this work he found the manuscript written by John Harrison in 1730 setting out his development of the grid-iron pendulum.

The Company, in connection with the Festival of Britain, issued a small booklet written by Baillie entitled *The Development of Time Measurement*. In this all the line drawings were made by him, and at the exhibition large scale models of the various escapements were made to his design by the apprentices of the British Tabulating Machine Company.

Baillie was a keen motorist and owned his first car about 1900. He was for many years a judge at the official trials of the Royal Automobile Club.

He joined the Physical Society in 1903 and was one of the Founder Fellows of the Institute of Physics in 1920.

As will have been gathered from the above, Baillie was a most versatile person: a first-class engineer, a great worker and an excellent linguist; with it all he was modest and sincere with a great capacity for friendship.

He died at Letchworth on the 19th November 1951, leaving a widow and a married daughter. R. S. W.

ALLAN HITCHEN FERGUSON

ALLAN HITCHEN FERGUSON was, in appearance, not unlike Samuel Johnson, though without the coarseness that seems to have been a prominent feature of the Great Cham of literature. With a height of about 6 ft. 6 in. he weighed over 21 stone, which implies a reasonable proportion between height and girth, for (on a cube law) it corresponds to a weight of 12½ stone for a man of 5 ft. 8 in. It meant, however, that he must of necessity be noticed in any company in which he found himself, and it was not easy for him to walk along the streets without having people turn round to look at him.

He was a man of very wide interests, which included the classical languages, parts of the history of physics and, appropriately enough, Johnsoniana. Like the earlier Doctor, he was much weaker on the Teutonic languages than on those of Ancient Greece and Rome. I well remember the Society receiving an address in Latin, informing us that some University was about to celebrate one of its centenaries and asking us to appoint a delegate to the celebrations. This was agreed, and a member nominated, and the question of an address for him to present to the University from the Society arose. Within a few hours, Ferguson had produced a flowery piece of Latin oratory (which was subsequently vetted by a Professor of Latin, who found nothing to correct) and when someone expressed surprise he merely said: " We haven't all forgotten our Matriculation Latin, you know ".

This versatility was evident, too, at the examinations in the University of London, where in later years he examined in Modern History (often in association with F. J. Weaver, late Headmaster of Braintree) as well as in Physics.

Ferguson was born at Entwistle on 11th May 1880 and spent his boyhood there, then at Wyke and Blackpool and finally in Preston. It was here that he started a course of training as a pharmacist's assistant and began to study at the Harris Institute for a London Science Degree. He entered the University College of North Wales, Bangor, as an Exhibitioner in 1902 and duly took the London Honours degree in Physics about 1904.

On graduation he became assistant lecturer and senior demonstrator in Physics at Bangor, where he remained until 1919. During this period he became interested in the mathematics of flight and, with E. H. Harker, wrote a book, *Aerial Locomotion* (with a foreword by Professor G. H. Bryan), which was published in 1911 by the Cambridge University Press in their series of Science Manuals. He took the M.A. (Wales) degree in Mathematics and his London D.Sc. in 1915 for research work in Surface Tension. His period at Bangor of course included the whole of the first World War, during which depletion of the staff left him for two years in sole charge of the Applied Mathematics division; in addition, for three sessions he had also to be the lecturer in Electrical Engineering. Even with this load of work he found time to be interested in hydro-electric development in Wales, and was an active member of the committee dealing with this matter. He was one of those sent to give evidence to the Water Power Resources Committee of the Board of Trade.

In April 1919 he took a post as Lecturer at Manchester College of Technology, but only stayed there for two years, after which he migrated to London, and in October 1921 became a lecturer in the physics department of Queen Mary College, at that time called East London College. He spent the rest of his career here, first as senior lecturer and, from 1934, as associate professor.

Immediately on coming to London in 1921 Ferguson joined the Physical Society, with which he was to be so intimately connected for more than 20 years. He was elected to the Council in 1925, and acted as Papers Secretary from 1928 until 1938. He was thus in office during many of the main events of the inter-war period, beginning with the amalgamation of the Optical Society with the Physical Society in 1932. He was, if not the prime originator of the idea of the *Progress Reports*, at least one of its main protagonists; he pushed the idea, against the opposition of those who thought it would involve financial loss, wrote to prospective authors, and himself edited the first few volumes. As he had to explain to the authors that long articles, involving a lot of hunting for references, were wanted, and that no payment was proposed for the first issues, his task cannot have been easy. At that time the office staff was small and the Society had no Secretary-Editor; the *Proceedings* were edited by Captain C. W. Hume, who devotedly gave his spare time to this work for many years, but naturally could not take on this additional task. Ferguson had a particular wish to include in each annual volume a report on the teaching of physics, a subject he had much at heart; the writing of it usually fell to him, and his report, though rather incongruous, was scholarly and often provocative. Ferguson also played a great part, with Ezer Griffiths, in organizing the International Physics Conference of 1934, which coincided with a meeting of the Symbols, Units and Nomenclature Commission of the International Union.

During his term as Papers Secretary and President of the Physical Society it became advisable to reduce costs and a change of printer was advocated. Ferguson had been appointed an Editor of the *Philosophical Magazine* in 1937 and had subsequently, in 1938, become a Director of the firm of Taylor and Francis, the owners of the *Philosophical Magazine*. At the suggestion of Wilfred Jevons, Taylor and Francis were asked to tender for the *Proceedings* and it was largely due to Ferguson that very satisfactory rates were obtained.

These inter-war years were, however, not without troubles for Ferguson. He had been having trouble with his eyes for some time, and in January 1933 he underwent an operation for cataract, but he was not long in enjoyment of the better vision which resulted. He noticed a blurring of things in the late summer of 1934, and was able to follow and report a progressive deterioration. He was in fact suffering a retinal detachment, for which he was operated on in October 1934. I remember vividly my visit to him in Moorfields Eye Hospital. He was under strict injunctions not to move his head, so that as one entered the room one saw the massive frame lying on its back, and a voice directed towards the middle of the ceiling asked who it was. He had been very interested in the operation (he was not given a general anaesthetic) and was vastly amused at the idea of electrical soldering applied within the human eye.

After this operation he recovered his vision to a great extent, but he was always a great anxiety to his friends during his visits to London. We had to pilot him about buildings, tell him where to expect a step, and guide him to his vehicle (it had to be a taxi rather too often for a modest pocket), yet he was always cheerful, ready with an apt quotation, and master of the subjects which he had come to discuss.

I do not know whether it was a deliberate choice, made when he came to London, or whether it was gradually forced on him, but from the age of 40 Ferguson devoted himself to those tasks of organization and committee attendance which are so necessary but so shunned by most scientists. Probably his combination of efficiency and of readiness to turn his hand to tasks that needed to be done made him accept first one post and then another till he had no option but to continue. I have sketched his connection with the Physical Society from 1921 up to the end of the inter-war period (he gave up the Papers Secretaryship in 1938), but he was equally active in other fields of physical organization. He joined the Institute of Physics as a Fellow in 1928, and in the same year began a term of service on the Board. In accordance with the rules he had to stand down in 1933, but he was back on the Board in 1934 and remained there, as member and then as Vice-president, until 1943. During this time he was active in the cause of unity, or at least of co-ordination, between the Institute and the Society, but in this regard he was before his time. There are still some who are active in this cause, and they will have their success some day, if only because logic demands it, but the time is not yet.

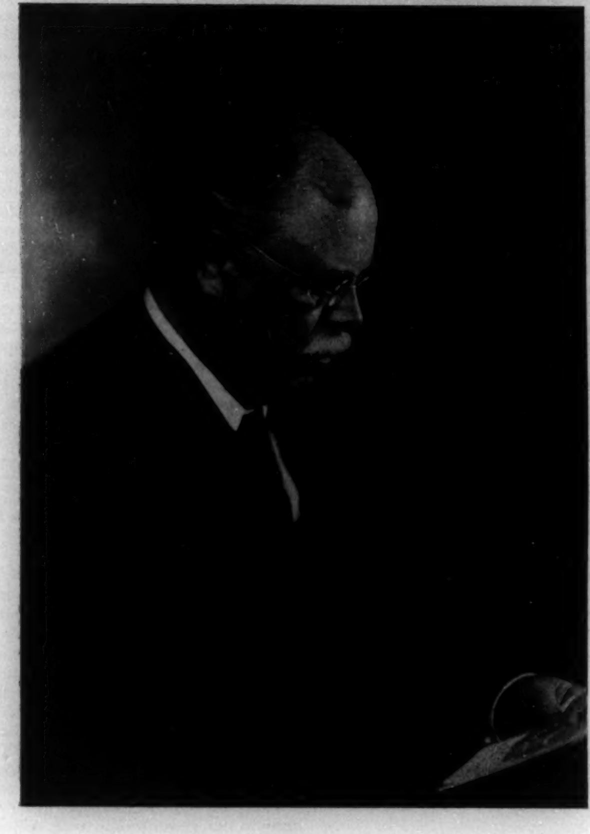
He was a member of the Faraday Society and became a member of its Council (1936–1937) and a Vice-president (1937–1943, with a statutory break of one year); he was prominent as a reviewer of books and referee of papers and, from 1947 until his death, was an Honorary Life Member of the Society.

I do not know just when he became Recorder of Section A of the British Association, but he held the position when I first joined the organizing committee, and I was staggered by the energy he put into the task of making the programmes for the annual meetings coherent. When the Association met at Blackpool in 1936 Ferguson took a great delight in being back on his own home ground, and had made it quite clear that the presidency of Section A at a meeting in that town would give him intense pleasure ; so that in 1936 he was prevailed on to hand over the post of recorder to Ezer Griffiths and himself became President of the Section, a well-deserved honour. At that time, he became also General Secretary of the whole Association, and retained this duty until 1946.

Just before the war, then, Ferguson had been Papers Secretary of the Physical Society, and had begun to take a major part in the editorship of the *Philosophical Magazine*. This did not end his part in the publications policy of the country at that time. It was in January 1939 that he began work as an Associate editor of *Nature*, and he carried this on into the early years of the war, during which time the proposal for a new journal, *Endeavour*, was mooted. He naturally became a member of the editorial panel of this publication, too.

But it is now time to consider the period of the second World War. Ferguson, on handing the Papers Secretaryship of the Physical Society to me, became President for the two-year period 1938 to 1940, but the outbreak of war presented the Society, like everyone and everything else, with new problems, and it was thought better, at the end of his normal term, to ask him to carry on for a further term, rather than to change Presidents in those early critical days. He therefore filled the office until 1941, the only President in the history of the Society to serve for such a term. The strain, with all these outside activities in addition to his work at College, must have been terrific, and I often feel that we may be in some degree responsible for the afflictions which he suffered soon afterwards. He slipped in boarding a bus and fell heavily, knocking his head; he was very badly bruised but he made light of the matter at the time. However, a type of paralysis gradually made itself manifest, which in 1942 was diagnosed as *myasthenia gravis*. It affected his powers of speech, and at first made it difficult to follow him, but later one became accustomed to it; perhaps too he gained more control. Moreover, he was given tablets which enabled him to gain temporary control for a lecture at college or any special occasion, and when he broadcast in a B.B.C. programme he was quite clear.

During the war Queen Mary College was evacuated to Cambridge, and Ferguson had the joy of lecturing in the Cavendish Laboratory theatre and of mixing with the Cambridge physicists. He retired from his University post in 1945, and was made a fellow of Queen Mary College in 1946. He was still very busy, and made frequent visits to London to attend various committees, remaining a member of the Editorial Board of the *Philosophical Magazine*, and in 1948 he prepared the special Commemoration volume, in which his knowledge of eighteenth century science shone through the whole plan. But his condition gradually became worse. Somewhat later his condition was aggravated by Parkinson's disease, and in 1949 further complication due to inflammation at the base of the oesophagus developed. From that time until his peaceful death on 9th November 1951 he lived on a liquid diet, devotedly tended by his wife Nesta, a former student of his, daughter of Mr. John Thomas, former Vice-Principal of the Naval College, Bangor, whom he had married in 1919. After his paralysis, it was in Nesta's handwriting that his characteristic letters used to arrive, for he did not allow such a minor matter as bodily weakness to interfere with his interests and activities, and she certainly would not let him give up if it were merely a matter of writing letters. Ferguson's son, John, had a brilliant career at his University, in which Allan took real pride, and their home was truly a happy Christian one.



Allan Ferguson.

Ferguson always preferred to live away from his work. When I first knew him he lived in Ongar, but as London spread he felt that this was not far enough out, and he moved to Bishop's Stortford. At the time of the move he told me how many tons of books had to be packed and crated; I think the number was five, but cannot be quite sure. At any rate, shelves were put in all the rooms for their reception. He used his journeys for reading, and in the Ongar days he had a spell of interest in detective stories, of which he reckoned to read one a day in the train. But this was not all his reading. He had, as has been said already, a deep interest in eighteenth century literary life, and all this was not to the exclusion of his scientific reading. He was an expert in classical physics, particularly surface tension, and knew a surprising amount about modern developments.

This was fittingly recognized by the Editors of the *New Chambers' Encyclopaedia*, who asked him to organize the articles on physical subjects. It reminds me that he must be one of the few people to review the *Encyclopaedia Britannica*; when the new, post-war edition came out he reviewed it for our *Proceedings*, not confining himself to the scientific articles, but commenting on its treatment of other subjects, and pointing out that it had no reference to the Papworth settlement for the tuberculous.

In politics Allan Ferguson was a Liberal, of the Manchester school, but he had a good deal of sympathy with the Labour movement, and an appreciation that there was much that is valuable in the doctrines of the traditional enemies of the Liberals, the Conservative party. He believed very strongly in the League of Nations, and was a leading light of the local branch of that organization. The failure of the League when its testing time came was a source of grievous disappointment to him.

Thus in his last six years, after his retirement, he had, despite his bodily discomforts, some consolations: a devoted wife, helping him to keep up his interests in the world of physics; a clever, loving son of temperament not unlike his own; he had his books, though he could not always read very much, and he had his correspondence with old friends. Letters, and visits from them, were a source of great consolation to him, and he was always more energetic after a group of scientific colleagues had been to see him. The end was a peaceful one, and the service at Bishop's Stortford preceding his cremation was attended by many of those who had loved and honoured him. J. H. AWBERY.*

* I should like to express my appreciation of the help which Dr. W. Jevons has given me in making this a less unworthy account; he has reminded me of some incidents which had slipped my mind and told me of others unknown to me which I am glad to see placed on record.

MEYER FINN

MR. M. FINN died on 8th February 1952 in his eightieth year; he had been a Fellow of the Physical Society for some fifty years. He was born of Jewish parents in Bialastock on the Russian side of the Polish border and managed to obtain a place in the Russian Gymnasium although only one per cent of the available places were then allocated to Jews. He came to England when he was seventeen and entered Armstrong College in the University of Durham. In three days he learnt enough English to attend lectures and finally became a distinguished mathematical scholar of the University, holding a Junior Fellowship for a time. Mr. Finn went to Southend in 1897 to take up work at a small school for boys and girls which eventually became Southend High School for Boys. It was during a Practical Physics Examination carried out for the Oxford and Cambridge Joint Board many years ago that I came into contact with him and was very much impressed by the fact that he had designed nearly all his electricity and magnetism experiments to work from the direct current mains. These experiments were described in his book *Electricity and Magnetism*, published by Bell, now, unfortunately, out of print.

As well as being Senior Master in the day school he was in charge of evening classes at the Southend Evening Technical Institute, where he trained many students for the London B.Sc. Degree, and organized comprehensive curricula in commercial subjects, cookery, dress-making, etc. He retired at the age of sixty-five, but during the war returned to teach Physics to sixth form boys of St. Paul's School and the Dagenham Secondary School.

Immediately after his arrival in England Finn became a naturalized British subject and was equally proud of his Jewish birth and his British nationality; he married an Englishwoman and they had eleven children, of whom seven are in this country, three in the United States and one in Australia. His wife died in December 1946 soon after their golden wedding, but he lived to see his great grandson. L. F. B.

JULIUS HARTMANN

By the death of Julius Hartmann on 6th November 1951 Danish science and technical research has suffered a great loss, and the Technical University of Denmark (T.U.D.) has been deprived of one of its most active teachers.

Hartmann's whole career as a scientific worker was intimately connected with and devoted to the development of his Laboratory of Technical Physics at the T.U.D. Already as a young student he began scientific work as Professor K. Prytz's assistant in the old Physical Laboratory. In 1914 he became a lecturer (Docent) of Physics, and in 1929 he was made a professor of Technical Physics, which latter discipline he had introduced as a new educational subject of the T.U.D. Ten years later a well-equipped laboratory—thanks to Hartmann's indefatigable efforts—was established in the new buildings then erected for the T.U.D., and here he spent the last years of his very active lifetime.

Hartmann's scientific work was mainly centred on problems raised by and connected with a series of inventions and discoveries most of which he had made during his younger years. Here may be mentioned his jet-wave commutator and rectifier for alternating current, which is described in a paper *Nye Ensrettere og periodiske Afbrydere* (1918), for which he obtained, as the first in Denmark, the Dr.techn. degree at the T.U.D.

Hartmann tried, with more scientific and technical than economic success, to develop the jet-wave rectifier into an industrial apparatus. In two big monographs, *The Jet-Wave Rectifier* (1929) and *The Large Capacity Jet-Wave Commutator*, I–II (in collaboration with L. Løgstrup Jensen, 1944 and 1946) and a number of smaller papers, Hartmann has given the results obtained by this work. Many new and important observations and inventions relating to fluid flow were made, such as the ‘ Plate-Jet ’ and the electrodynamic mercury-pump.

When trying to measure the pressure-distribution in an air-jet of supersonic velocity by means of a Pitot-tube Hartmann observed pressure fluctuations in the Pitot-system. A closer investigation of this phenomenon led to Hartmann's construction of a generator for very intense sound-waves, described in *The Acoustic Air-Jet Generator* (1939) and now generally known as the ‘ Hartmann Acoustic Generator ’ and mentioned in all modern textbooks of acoustics. In many laboratories it has been used as a most convenient and practical source for high-intensity sound waves.

Hartmann has described many interesting and important phenomena produced by use of his generator, such as the association and precipitation of particles in a colloidal suspension and, under certain other conditions, the opposite effect, production of emulsions. Certain biological effects, as, for instance, disinfection of liquids containing microbes, have also been observed.

For the detailed investigation of the structure of a pulsating air-jet Hartmann, in collaboration with F. Larris, developed a special Schlieren method which gave very nice photographic results.

As a teacher of physics at the T.U.D., where the Danish engineers (Civilingeniører) are educated, Hartmann's endeavours were directed mainly on the experimental work and especially on the techniques of measurement and evaluation of observations. He wrote several textbooks on these subjects; the last of these came from printing just a few days before his death.

Personally Hartmann was a kind and modest man, who preferred work to most other forms of human enjoyment of life. His many friends and admirers in his own country and abroad deeply regret his death.

R. E. H. RASMUSSEN.

CHARLES HERBERT LEES

By the death of Professor C. H. Lees the Physical Society has lost its oldest past president and the world of physics will mourn the departure of one of the ‘old guard ’ of the last generation. He was 88 years of age and died on September 25th 1952. Lees was born in Glodwick, Oldham in 1864, the second of three sons of Jane and John Lees. John Lees was one of the gold seekers who went to Australia in the '50s. He and his partner found the ‘ Leg of Mutton ’ nugget at Ballarat. With the proceeds John Lees founded the Ballarat building in Oldham and C. H. Lees as a young man was always interested in the upkeep and repair of the property.

Lees left school at the age of 14 and went as a clerk to the Gas Company's office in Oldham. He attended night school and gained a scholarship to the Owens College, Manchester. When he took his degree he was a Derby Prizeman and elected Bishop Berkeley Fellow in 1888 : on an average about three of these were awarded each year from 1881 to 1895, after which they were discontinued. They were open to all Faculties and Lees was one of only five physicists elected to fellowships during the 16 years. He studied at the University of Strasbourg for about two years and then spent a few months teaching at a school. His Strasbourg professors were Kundt in physics and Christoffel and Rye in Mathematics. In 1891 he returned to Manchester and was appointed senior assistant lecturer and demonstrator in Physics in the Owens College and became lecturer and assistant director of the Physical Laboratories of the University of Manchester in 1900. He worked under Schuster. Until his marriage in 1902 Lees spent much of his holiday time walking and geologizing in Lancashire and Yorkshire, often accompanied by his younger brother. He left Manchester in 1906 to become professor at Queen Mary College, then East London College. Just before he left Manchester he went to South Africa as secretary of Section A (Physics and Mathematics) of the British Association. He served Queen Mary College until 1930, retiring with the title of Emeritus Professor of Physics in the University of London. He was Vice-Principal of the College from 1918 to 1930 and a representative of the Academic Board on the Governing Body from 1916 to 1930. It is certain that the Principal, J. L. S. Hatton, relied a great deal on Lees' judgment in college administration and academic matters. In his earlier years at the college he had one lecturer to share the work and there were evening as well as day classes. By 1929–30, his last session, his staff had increased to four lecturers and the evening classes had been abolished. Throughout the time Lees took a large share in the lecturing to Intermediate, General B.Sc. and Special Honours students.

It was in 1907 that the College first obtained recognition as a school of the University of London for an initial period of three years. This was extended for a further five-year period in 1910 and finally without limitation in 1915. Lees' part in developing the Physics Department and his general influence on college affairs must have contributed much to this.

He was one of the first three to be elected Fellows of Queen Mary College when it received its new charter in 1934. For many years Lees served on the Kent Education Committee and the Delegacy for Managing Goldsmiths' College.

Lees' major contribution to physics was in the field of thermal measurement. During his stay at Strasbourg he carried out some experiments on thermal conductivity by the 'periodic' method first used by Ångström. He found that the variation of temperature along the bar could not be represented by the expression deduced by Ångström from Fourier's equation for the flow of heat along a bar. On his return to Manchester he made a theoretical study of the problem and published a paper "On the Law of Cooling, and its bearing on the theory of the Motion of Heat in Bars". This was published by the Manchester Literary and Philosophical Society 1889–90.

Three papers were published in the *Philosophical Transactions of the Royal Society* in 1892, 1905 and 1908. The last of these, on "The Effect of Low Temperatures on the Thermal and Electrical Conductivity of Certain Approximately Pure Metals and Alloys," constituted the Bakerian Lecture for 1908. (In this latter work he used resistance thermometry and acquired first-hand knowledge of the use of this method of measuring temperature by working with E. H. Griffiths.) In these papers Lees displayed his mastery of mathematical technique when dealing with the calculation of correction terms. Lees' method for the measurement of the conductivity of materials in the form of discs is now usually found in modern textbooks.

His interest, however, was very wide as will be seen from the titles of some of his other papers.* In 1901 he published a paper in the *Proceedings of the Physical Society* " On the Viscosities of Mixtures of Liquids and of Solutions." In 1929 he contributed to the Society a paper on " The Free Periods of a Composite Elastic Column or Composite Stretched Wire". This arose out of some investigations carried out at the Safety in Mines Research Stations at Sheffield, Lees serving on the Safety in Mines Research Board. His presidential address to the Physical Society was " On the Mechanical Equilibrium of a Sphere of Gravitative Fluid." He also contributed papers on a.c. bridges and on fluid flow through pipes. Lees took a delight in the geometrical or graphical representation of results.

* A complete list of his published papers is available at the offices of the Physical Society.

The writer's first contact with Lees came early in the formation of the Engineering Committee of the Food Investigation Board, now the Food Investigation Organisation. Sir William Hardy, with his sure instinct for selecting the right man, persuaded Lees to serve on this Committee, which had Sir Alfred Ewing as its chairman, and also on the Insulation Sub-Committee.

The first report they produced was on the literature of refrigeration; this was followed by a critical study of insulated rail vans. They drew attention to the disadvantages from the thermal insulation aspect of the considerable number of metal bolts and metal components penetrating the insulation. The point of their criticism will be appreciated when it is remembered that the conductivity of iron is about 1000 times that of cork. For a period of over twenty years the Engineering Committee continued to function. It supervised the work of Professor Frewen Jenkins at Oxford on the properties of refrigerants, and it encouraged work on insulating materials, humidity and various other questions at the National Physical Laboratory ; under its auspices the overseas transport of fruit and meat was investigated. One of its efforts in full scale research was the purchase for the Board of a Thames barge, insulating the holds with various materials and equipping it with a refrigerating plant which could be operated from the engine when this was not required for propelling the barge. As an experiment the venture was a success and a profit was shown, but the conservative attitude of those engaged in this trade retarded its adoption.

Lees was president of the Physical Society from 1918 to 1920, and was also a vice-president of the Institute of Physics. His hobby was handicrafts, and to within a few days of his death he occupied his time doing carpentry. He was a man of great patience and was not to be deviated from the course he had set himself by difficulties when carrying out a piece of research. When dealing with research students, if unable to give immediately the complete answer to a question he never failed to throw some light on the problem or make valuable suggestions for the solution. He endeared himself to colleagues and students by his great charm and sincerity and we mourn the loss of a staunch friend. He is survived by his widow and five children, all of whom are associated with scientific work. His eldest daughter is the wife of Professor G. L. Brown, C.B.E., F.R.S. EZER GRIFFITHS.

WILLIAM LETHERSICH

WILLIAM LETHERSICH, who died on 2nd March 1952 at the age of 46, gave an example of achievement in the face of physical handicap. Educated at the Tottenham County School, he entered Queen Mary College (London) in 1925, securing in 1928 Second Class Honours in Physics, in spite of his indifferent health. After spending a year on research in the Physics department he transferred in 1929 to the Electrical Engineering department under Professor MacGregor-Morris, where he introduced the double balanced calorimeter method of measuring dielectric losses at high frequencies based on the principle of the rate of temperature rise. The method is still in use. In 1930 he joined the staff of the Electrical Research Association, where he remained until his death. At first he worked with Dr. H. G. Taylor (now Director of the Welding Research Association) on the heating of lamp fittings and developed the Schlieren method of displaying and analysing hot air currents for this purpose, inventing certain ameliorative measures. His methods have been widely developed and his improvements are incorporated in modern design. Regarding the risk to wooden poles for overhead lines with abnormally wet heart wood, in the event of insulator failure, he elucidated the mechanism and showed the risk to be slight and capable of avoidance by a very simple expedient. He then began his main scientific work, which was on the rheological properties of insulating materials. He gave a definition of plastic and brittle failure of bitumen, together with a quantitative relation between tensile strength and rate of loading. His greatest achievement, on which he was working when his last illness began, was probably his application of the theory of linear phenomena to rheological properties. He showed that the behaviour of a large number of materials under shear stress could be quantitatively expressed in terms of a simple spectrum of relaxation times, over a range of time intervals from the order of 10^{-5} to 10^{4} sec, 9 decades. In this work he produced several ingenious experimental devices, described in his publications and exhibited at the Physical Society's Exhibition and elsewhere. Full recognition was rather slow in coming to Lethersich on account of a rather obscure literary style, but in the last few years his reputation for theoretical acumen and elegant experimentation had grown rapidly. He was a Fellow of the

Institute of Physics and an Associate Member of the Institution of Electrical Engineers, as well as a well-known member of the Rheologists' Club. He published several papers in the *Proceedings of the Physical Society*, *Journal of Scientific Instruments*, *Proceedings of the Institution of Electrical Engineers* and *British Journal of Applied Physics*.

Lethersich was not just a scientist, he was an accomplished musician and a man of religious principles, esteemed by a wide circle of friends. He was a bachelor who devoted himself to the service of his community. He gained first prize for the organ at the Stratford Music Festival and acted as Organist and Choirmaster at Queen's Hall Methodist Church, Hayes, for 17 years. Frequently he took duty for his former teacher, Mr. Edgar Humphreys, at Bow Church. The heart trouble which led to his death was a serious handicap throughout his life. In spite of it he made permanently valuable contributions to all the scientific fields of his professional endeavour, in the arts he was admired and as a friend and teacher he was loved.

S. WHITEHEAD.

FREDERICK HENRY NEWMAN

FREDERICK HENRY NEWMAN was born on 8th June 1891, and was educated at the Bath Technical College (Secondary School) from 1904 to 1911, and at the Royal College of Science, South Kensington, from 1911 to 1914 as the holder of a Royal Scholarship. In 1913 he graduated with First Class Honours in Physics and obtained the Associateship of the Royal College of Science. He came under the direct influence of Lord Rayleigh and Professor A. Fowler as a research student during the session 1913 to 1914. He then proceeded to University College, Aberystwyth, as Lecturer in Physics, and in 1915 carried out some research on X-rays, in the course of which his left hand was very severely burnt, which resulted in a wound that never healed. This was a source of much discomfort for the rest of his life. He obtained a commission in the Forces and served in a technical capacity from 1915 to February 1919, when he retired as Captain.

In March 1919 he was appointed to the only lectureship in Physics in the former Royal Albert Memorial College in Exeter. He then had but a few students, and of the three whom he prepared for the London B.Sc. (Special) degree, two obtained First Class Honours in Physics. This was an augury of the excellent work he was to do at Exeter. He remained a lecturer until 1922 when the College was placed on the Treasury List by the University Grants Committee as the University College of the South West, and in it he became the occupant of the newly-created Chair of Physics. During his tenure his Department grew in every way, so much so that its present teaching staff of six is fully occupied. He was largely responsible for the design and equipment of the Washington Singer Laboratories, opened in 1931, of which he became Director.

Professor Newman held the degrees of D.Sc. and Ph.D. of the University of London (he was the first external student to receive the London Ph.D. in Physics), the diploma of membership of the Royal College of Science and Technology (D.I.C.), was a Fellow of the Institute of Physics, of the Physical Society, and of the Institute of Radiology. He was a member of la Société Française de Physique and A.M.I.E.E.

His research work, which was begun at the Royal College of Science, first dealt with the absorption of gases by various metals that had been deposited on the cathodes of electric discharge tubes. This work he resumed at Exeter after World War I, and it subsequently developed into extensive spectroscopic investigations of vapours of the alkali metals and mercury and of the gases, hydrogen and argon, contained at low pressures in discharge tubes at various temperatures. His interest in physical research never waned, although his output of published work in recent years inevitably became less owing to the encroachment on his time of administrative work. Newman also directed the researches of numerous students for whom he chose themes other than Spectroscopy, and the results of their work have been published under their names, so that his influence in the discovery of new knowledge extends well beyond his own particular field of research.

Dr. Newman was the author of three monographs on Physics, viz. *The Production and Measurement of Low Pressures* 1925; *Electrolytic Conduction* 1931; and *Recent Advances in Physics* 1931. In collaboration with one of his former students, V. H. L. Searle, he wrote a standard text-book on *General Properties of Matter* which passed into its fourth edition in 1948 and has since been reprinted several times.

On the outbreak of World War II he was both Chairman and Secretary of the Joint Recruiting Board of the South Western Area, and it was not until 1947 that he relinquished the Secretaryship. On the cessation of hostilities he undertook yet more voluntary work and became Chairman of the Exeter Disablement Advisory Committee of the Ministry of Labour. For many years he was a Scientific Consultant to the Exeter City Police, and in this capacity gave many lectures at police conferences on Scientific Aids in the Detection of Crime and acted as an expert witness in many cases. During the recent war he acted as adviser to one of the Services. He also organized intensive courses for Radio-bursars of six months' duration, of training in Radio Communication over a period of three years, and towards the end of the war he directed two one-year courses for Cadets of the Royal Corps of Signals. Recognition of this public service came in the form of the C.B.E. in the New Year's Honours List of 1949. Since then he was appointed as one of the three West Regional Scientific Advisers to the newly-created Civil Defence Service.

Following the resignation of Dr. John Murray from the Principalship, Professor Newman undertook the dual task of Acting Principal of University College, Exeter, with nearly a thousand students, and that of running his own Department. At the present stage in the development of the College the work entailed by the combined posts was colossal, but Newman managed to cope with the task with an efficiency which characterized everything he undertook.

His powers as a teacher and research worker were considerable. He was extremely popular with his students, both present and past, who frequently sought, and were benefited by, his advice. By nature he was quiet and reserved. Despite the very active life which he led he was never physically robust, and owing to a life-long disability, coupled with the x-ray injured hand, he was frequently in great pain, although he took especial care not to allow the fact to become apparent to others.

His death on 15th April 1952 was comparatively sudden, and in his passing the University College of the South West and the City of Exeter have suffered a great and severe loss. He is survived by his widow, a daughter and a son.

H. T. S. BRITTON.

MALCOLM MACLEOD NICOLSON

MALCOLM MACLEOD NICOLSON, Lecturer in Physics at Leeds University, was killed in an accident at a railway station near his home on 21st December 1951. He had been to meet some friends who had been delayed by the thick fog, and whilst leaving the station by a private crossing was struck by a passing train and killed instantly.

He was born in Skye on 6th July 1919, but his family later moved to Lancashire and he entered Manchester University in 1936, graduating with First Class Honours in Physics in 1939. He worked as a research student under Professor Hartree and took an M.Sc. in the following year.

After a short period of service in the Royal Navy, followed by work on radar development, he was appointed in 1941 as lecturer in the Gunnery and Mathematics department of the Royal Military College of Science. In 1945 he went to Cambridge, where he worked under Dr. Orowan on surface phenomena in ionic crystals, work for which he was awarded a Ph.D. in 1948; whilst at the Cavendish Laboratory he also carried out theoretical work on thermionic valves under the direction of Professor Hartree.

Nicolson came to Leeds in October 1948, and in the brief three years he was there he made himself an integral part of the life and work of the Physics Department. Although primarily a theoretical physicist, he took a very active interest in the experimental work in the teaching laboratories, and his lectures were marked by the thoroughness with which he covered his subjects. At the time of his death he had nearly finished writing a book on mathematics for physicists. Arrangements are being made for the completion and publication of this book.

Occupied though he was in teaching and writing, he was generous to a fault in the time he would devote to problems brought to him by colleagues and research students. The loss of a young scientist of great promise, able and enthusiastic, with high standards and sound critical judgement, is a heavy one, and his many friends will sadly miss a delightful companion, lively and forceful in discussion, humorous and kindly, and utterly unselfish.

In 1942 he married Miss Phyllis Lockett, whom he leaves with two young sons.

R. S. T.

REVIEWS OF BOOKS

A Hundred Years of Physics, by W. WILSON. Pp. 319. (London: Duckworth, 1950.) 21*s*.

It is an unforgivable remissness to delay the review of scientific works, particularly one of Physics, for their value is usually strictly temporary and most are out of date before they are printed. My excuse in this case is that here we have a book which will be of permanent value to the students of physics and, as it deals with a hundred years of the subject, will not suffer from waiting two or three for a review.

Most books on the history of science tend to be a catalogue of dates and discoveries interspersed with biographical details more or less relevant to them. They are necessarily unintelligible to anyone not knowing the subject thoroughly. Professor Wilson has not followed this tradition. Indeed he has written on the last hundred years of physics hardly bringing in history at all. It is idle to look in this book for a critical account of the development of physics from 1850 to 1950 or to learn anything about the men who were responsible for its chief advances. Nor will anything be found of the applications of physics or its contribution to the present atomic age, nor even to the influences of industry or war on the advance of physics. For all such information we must look elsewhere in the wide though scattered literature on all these subjects.

What we will find in Professor Wilson's book is a description, on a level suitable to a first year university student, of the actual advances in physics themselves, arranged in a logical rather than historical order. Thus it provides a new, attractive and, at the same time, sound introduction to physics, not as a set of dogmas of accepted knowledge, but as an interlinked series of successive approximations plainly unfinished. He follows out three main themes: thermodynamics, including the derivation of the first and second laws; the electromagnetic theory of light; and modern physics of the quantum theory and relativity. Anyone following his chapters, which are copiously supplied with references to original papers, will be able to learn physics intelligibly set out as it was built up.

There is, in addition, a most valuable bibliography. Of particular value are the simple descriptions, experimental and mathematical, of the main advances. They can be used as they stand as a basis for teaching.

Of course the picture is not complete; for brevity's sake branches and links, often important ones, are altogether omitted. There is a notable lack of description of the purely technical advances which made the scientific discoveries possible. The index does not contain the words vacuum or valve. But for what it sets out to do *A Hundred Years of Physics* is pre-eminently successful. It should find its place in every teaching laboratory of physics and the wise lecturer will want to look at it again each time he prepares his courses.

J. D. BERNAL.

Science in Progress, Sixth Series, edited by G. A. BAITSELL. Pp. xv+322. (New Haven: Yale University Press; Oxford: University Press, 1949.) 40*s*.

The progress of science moves with an accelerating pace, but the communication of the results is as slow as, if not slower than, ever. Most of the essays which compose this book were delivered in 1947. The book was published in 1949, reached this country in 1950 and is being reviewed, owing to the inexcusable dilatoriness of the reviewer, in 1952. It would have been useless, even without that, to discuss these essays as the latest fruit of scientific advance. That they are worth reading now, and most of them are, is due to more intrinsic and time enduring factors. All are well and clearly written and, though subsequent discovery has added to the picture in many places, as records of achievement they still hold their own.

The authors of the physical articles, Smyth, Wheeler, Lawrence and Seaborg, are all renowned masters of their field in American science, as are those in the chemical and biological fields, Pauling, Zechmeister, Stanley, Dubos, Beadle, Sturtevant and Kellog. Most are the directors of vast new laboratories with the super apparatus which has been a magnet to scientists all over the world. What they have to say is the real matter of the history of science. Indeed, Smyth's article ***From X-Rays to Nuclear Fission*** is ostensibly history and interesting for the critical comparison of the science of 1942 with that of 1896.

" The chain reaction, the next and last discovery to be mentioned, was made under circumstances superficially very different—in secret, in a war laboratory, heavily financed by the United States Government. Even now, a complete quantitative description of the chain reaction has not been published. The beauty of Roentgen's paper or of those on fission was that anyone who read them could repeat the experiment. This is perhaps the most fundamental principle of science ; its methods of operation are so objective and its means of communication so precise that its results are reproducible. Each man can build on the other's work so that knowledge is advanced by a vast cooperative movement.

But we cannot work in this way when atomic bombs may be the reproducible result in a world that is politically unable to control them. Hence, we have to violate our fundamental principles and keep silent about numbers and dimensions." (Page 20.)

" Up to the time that Fermi and his group first made the nuclear reaction go, the uranium project was essentially scientific in aims and methods. December 2, 1942, represents the end of the epoch of atomic physics that began in 1895. Whether the effort of scientists in this period will ultimately benefit mankind or destroy it remains to be seen. The power of the scientific method has been amply demonstrated." (Page 21.)

This is certainly a fact, but its consequences inside and outside science have yet to be fully appreciated. Lawrence on *High Energy Physics* will long be worth reading. Though the achievement of a mere 200MEV in the Berkeley Synchro-Cyclotron seems now small beer in the Bevatron area, the story of it is like a new voyage of discovery, though it must have cost several hundred times what it did to discover America in the first place. In the same way, Seaborg's story of *Eight New Synthetic Elements* will stand, however many more have still to be described or concealed.

So much for the part of *Science in Progress* devoted to physics. In the remaining field the physicist can learn of the great unification of chemistry and biology in the study of enzymes, viruses and microbiology generally. Here the picture is naturally far more complex, but the methods are increasingly the quantitative approach that derives from physics, and advances are almost as rapid. They have, however, a way of being less secure partly due to an over-simplified carry-over of ideas from a different field, and it is probable that the biological section of *Progress* already needs more revision than the physical.

J. D. BERNAL.

College Physics, by F. W. SEARS and M. W. ZEMANSKY. 2nd edn. Pp. 912. (Cambridge, Mass. : Addison-Wesley Press, 1952.) $8.50.

The first edition of this book appeared in 1947, and the alterations made by Professors Sears and Zemansky include a completely new set of problems, and the addition of a number of fresh topics among which are surface tension and the derivation of lens and mirror formulae. The preface affirms that the emphasis is on physical principles. This claim is justified in that the groundwork of mechanics, electricity, and optics is presented clearly using elementary mathematics and without too much pure description. As in all elementary teaching, compromise has had to be made by appealing to intuition in some places, and merely stating results in others; but where the emphasis is most needed it is plainly evident.

The early chapters on statics and dynamics are well done, with many striking microflash photographs of different kinds of motion. Hydrostatics and properties of matter receive less detailed treatment (there is a confusion between elastic limit and yield point in the chapter on elasticity), and the five chapters on heat seem somewhat condensed. In the light section there is a good chapter on photometry, and the chapters on diffraction, polarization, and colour are outstanding for their detailed treatment and excellent illustrations. It is unusual to find so full and accurate a treatment of colour in a book of this standard. It should be mentioned that the sign convention used for mirrors and lenses is not one of the two recommended for elementary teaching by the Physical Society's *Report on the Teaching of Geometrical Optics*, being Case 3 of Group I in the classification there given.

Both the advantages and the difficulties of the modern approach to electromagnetism are made apparent. Granted that torsion-balance experiments are imperfect, and magnetic poles an artifice, these can at least be interpreted by the beginner in his own simple terms and related to the laws of mechanics. The problem nowadays is where to make a start which will evoke the same use of the student's grasp of physical principles, rather than his awareness of modern technical progress. Young people today probably lisp in atomic numbers, and it is proper to appeal to this interest; for teachers there is a moral to it all—you get no bread with one pith-ball! Yet it is curious that the scattering of α-particles should be given prominence by the authors as a direct verification of Coulomb's law to people who will not have reached the mathematics of central orbits. Again, the cathode-ray tube is used in introducing the idea of magnetic field. This is fair enough, but some rather elaborate manipulation of it is imagined in order to demonstrate the law of the action of field on current, which is the starting-point of electromagnetism. After this, the rest of the treatment proceeds quite elegantly, in terms of B. The vector H does not appear until the later chapter on ferromagnetism, and is sufficiently important to have been explained earlier. Incidentally, in the section on electromagnetic radiation it might have been well to state why H rather than B is concerned. This section of the book ends with a short chapter on alternating current, which summarizes the chief results without going into great detail, and gives a good outline of elementary electronics.

The book is well written and excellently produced. It can be recommended to physics teachers in schools, who will find the outlook of the chapters on electricity interesting and on the whole convincing. The brighter sixth-form pupil would gain a great deal from it also, but as with so many well-upholstered American books at this level, the chief difficulty about recommending it generally is the price. G. R. NOAKES.

French–English Science Dictionary, by LOUIS DE VRIES. Pp. xii+596. (London: McGraw-Hill, 1952.) 2nd edn. 55*s*. 6*d*.

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Professor de Vries has already done great services for workers in science, and one has come to expect his dictionaries to be competently prepared. It is thus no surprise to find this the case with the second, enlarged, edition of his *French–English Science Dictionary*. The ' enlargement ' takes the form of a supplement of terms used in aeronautics, electronics and the trio radar, radio and television.

The merest beginner need have no fears about using this work. The parts of many irregular verbs are given separate entries (' va ' and ' vais ' under ' V ', for example!), the gender of each noun is indicated, and even plurals such as ' chevaux ' are given.

The original volume contained 43 000 words. The supplement brings another 50 pages, and this is followed by 12 pages of abbreviations together with the full words and their English equivalents.

The words are chosen from agricultural and biological subjects as well as from the physical sciences. Nevertheless physics seems to be well represented, especially now that the supplement has been added. The book is well set out and pleasant to use.

H. H. HOPKINS.

Phase Transformations in Solids, by R. SMOLUCHOWSKI, J. E. MAYER, and W. A. WEYL. Pp. x+660. (New York: Wiley; London: Chapman and Hall, 1951.) \$9.50, 76*s*.

This book records the contributions of eighteen authors to a symposium on the subject of phase transformations in solids held at Cornell University August 23rd to 26th 1948. There are seventeen separate papers each in the nature of a review of part of the subject. The first six papers are devoted to theoretical studies under the titles: (1) On the General Theory of Phase Transitions by Laszlo Tisza, (2) A General Method for Imperfect Crystals and Phase Transitions by Joseph E. Mayer, (3) Crystallisation as a Cooperative Phenomenon by John G. Kirkwood, (4) Fundamental Aspects of Diffusion in Solids by Frederick Seitz, (5) Nucleation Theory by R. Smoluchowski, and (6) Crystallographic Aspects of Phase Transformation by M. J. Buerger. For the benefit of the general reader it would help if the first five of these papers could be related more closely than they are to the experimental work which follows. Buerger's contribution contains a classification of transformations relating the coordination of the atoms to the type of transformation involved and is a comprehensive review of the phenomena occurring principally in inorganic crystals.

The seventh paper deals with The Solid–Liquid Transition in Argon by O. K. Rice and the eighth with Transitions in Silver Halides by M. L. Huggins. The next three papers are devoted to silicates : (9) Phase Transformations in One-Component Silicate Sustems by F. C. Kracek, (10) Phase Transformations in Polycomponent Silicate Systems by J. F. Schairer, (11) Transitions in Glass by Woldemar A. Weyl. Papers (9) and (10) contain comprehensive accounts of the many and complicated phase transformations which occur in these systems. There seems as yet little possibility of predicting the nature and the conditions under which these changes will occur. The paper on glass is almost wrongly titled. The various continuous changes in properties associated with changes in composition and treatment hardly merit the description ' phase change '. Apart from this the paper is a valuable account of the changes which can be brought about in various glasses. Paper (12) is a short account of Phase Transitions in Ferro-electrics by Bernd Matthias. The remaining half of the book is devoted to metals. The subjects treated are (13) Transformations in Pure Metals by Charles S. Barrett, (14) Order–Disorder Transitions in Metal Alloys by Sidney Siegel, (15) Precipitation from Solid Solutions of Metals by A. H. Geisler, (16) The Eutectoid Reaction by Robert F. Mehl and Arthur Dubé, and (17) The Martensite Transformation by Morris Cohen. The section on metals and alloys is invaluable for anyone working in this field. There is a mass of well documented information covering all aspects of the many types of transformations which occur. The state of flux of knowledge is well shown by the new phenomena which are still being discovered by x-ray and other methods and no attempt at a final statement on many of the problems can as yet be made.

The organizers of the symposium are to be congratulated on their foresight and the publishers on the excellence of the production. As a reference work the book is bound to be of great value to many kinds of specialists for a long time. W. A. WOOSTER.

An Introduction to Mathematical Physics, by R. A. HOUSTOUN. Pp. x+262. (London : Blackie, 1952.) 25*s*.

The familiar pre-war text book of the same title which was first published in 1912 has been completely revised and extended in scope. In addition to chapters on attractions, hydrodynamics, heat conduction, wave motion, electromagnetic theory and thermodynamics, there are two new ones on quantum theory and relativity. As is to be expected in a book of this size, only the elementary parts of these subjects are included.

Where necessary, each topic is introduced by a short statement of the basic physical principles on which it rests, and, though these statements are rather over-simplified in places, they are in general sufficient introduction to the mathematical development which follows.

The reader is expected to have a fair knowledge of calculus, but the analysis required is derived at appropriate places in the text. Because of its extremely clear and straightforward presentation, the book will be of great assistance to physics students (for whom it was mainly written).

There are over 100 examples. Vector notation is not used. A. N. GORDON.

Servomechanisms and Regulating System Design, *Vol.* 1, by H. CHESTNUT and R. W. MAYER. Pp. xiii+505. (New York : Wiley ; London : Chapman and Hall, 1951.) 62*s*.

Being a relatively new field of study, much of the published work on this subject has so far appeared only in the proceedings of professional institutions. In such papers the method of analysis favoured by the particular author is dealt with to the exclusion of others. In the present book an attempt is made to bring aspects of several methods within the compass of a single volume. The authors have succeeded in the difficult task of presenting a coherent statement of design methods which reduce system design to a straightforward and unambiguous process.

The authors have clearly aimed at producing a readable book about a rather specialized technique and as a consequence the volume is considerable in size, as well as performance. It commences at a more elementary level than most readers who are able to follow it to its conclusion will feel is necessary. Discussion of the detailed characteristics of components and apparatus is not included in this volume, the primary concern being to establish a firm foundation of design procedure based upon the harmonic response method of analysis, including the vector loci methods and the plotting of the logarithm of the transfer function to a base of log frequency. A wide range of practical designs are given in the text and problems on each chapter are included which will assist the novice. The bibliography is excellent, containing over 100 references.

The book is dedicated by the authors to the advancement of engineering practice. One has the impression that they therefore devote attention to saying how things are done rather than why they are done. This may leave the reader in some doubt if the case he has in mind is not a straightforward one and in this field, more than others, special cases are rampant. The book caters particularly for the need of those who want to design adequate control systems of a classical type and provides graphs and devices for simplifying the work. An effort has been made to assist the beginner by the inclusion of some basic mathematical material. The authors have succeeded in their purpose of presenting a self-contained statement of current practice. J. WESTCOTT.

Mechanics and Properties of Matter, by R. J. STEPHENSON. Pp. x+371. (New York: Wiley; London: Chapman and Hall, 1952.) $ 6.00 (48*s*.)

In company with a number of other recent American publications this volume sets out to give a thorough course on its own branch of the subject, which assumes that the reader has already studied physics and the calculus during a preliminary year. It is evidently intended to form an introduction to the more advanced study of all branches of physics, for it is frequently stressed that a sound knowledge of mechanics not only underlies the later sciences of electricity, heat, light and even atomic physics but has in addition profoundly influenced their development.

The book covers much of what, in a British university, would be regarded as belonging to the Intermediate course, together with the substance of the first-year's work in an Honours course and a few topics which would often be deferred until a later stage. The fact that we normally have an examination at the Intermediate stage means that we should not expect to find the elementary and advanced material contained in the same chapter, as it is in this volume: for instance, the chapter in which simple moments of inertia are introduced and calculated closes with such topics as the precession of a gyroscope. Such a method of selection and presentation must necessarily narrow the appeal which the book makes to an English university student.

There is, however, a freshness of approach which is altogether exhilarating and admirable. The treatment of even elementary mechanics makes the reader aware right from the beginning that the Newtonian system is an approximation only, true enough at low velocities for practical purposes; and though the theory of Relativity is not developed formally, many of the important expressions derived from it are quoted in the text. Moreover, the treatment of mechanical problems generally gets away from what, for us, have become the conventional illustrations: the centre of percussion of a bar is illustrated from baseball instead of cricket, the collision of elastic spheres from the slowing down of neutrons in a carbon moderator, and the reader is made aware that he is living in the second half of the twentieth century even in his study of what has been known for two hundred years.

A possible criticism is the actual balance which is struck between pure mechanics and properties of matter. By comparison with the treatment of mechanics the sections on the properties of matter are somewhat cursory, and lacking in practical detail.

Throughout the book the mathematical argument is set out fully and clearly. Vector methods are used where it is felt that the presentation of physical principles is thereby clarified, for example in the general discussion of the rotation of a rigid body. The diagrams and the production generally are excellent, and the value of the book is considerably enhanced by a good selection of numerical examples to be worked by the reader. Even at the relatively high price of 6 dollars the book should find a market in this country. G. D. YARNOLD.

Comets and Meteor Streams, by J. G. PORTER. Pp. xi+123. (London: Chapman and Hall, 1952.) 28*s*.

The great displays of shooting stars, or meteors, towards the end of the 19th century stirred the popular imagination, and also gave evidence that meteors must be associated with comets in some way. The difficulties of investigating such transient phenomena, often obscured by cloud or moonlight at critical moments, sapped the interest of all but the most enthusiastic investigators and for the first 45 years of the twentieth century progress was rather slow. High expectations were raised by the great meteor expedition to Arizona which began in 1930 under the auspices of Harvard Observatory, but unfortunately the results served to deepen the schism which existed between those who believed in an interstellar origin for the majority of meteors and those who favoured a solar system origin. Against such a background it is easy to appreciate how the new post-war radio techniques infused new life and interest into the study of meteors and their origin.

By a felicitous circumstance one of the early provincial meetings of the Physical Society, held in Manchester in 1947, was devoted to the subject of meteors, comets, and meteoric ionization, and it was there that the book under review had its origin. The rapid advances made in the study of meteors since then have underlined the author's view that " it is impossible to consider any study of comets and meteors which does not involve the paths in which they travel ", and all associated with the subject will be glad that Dr. Porter's main emphasis in the book is on " the orbits of these bodies and the changes to which they are subject ".

The author has written within these imposed limitations, and his book is not for the reader who seeks to find a detailed account of the physical processes relating to comets and meteors. For example, the account of the structure and general features of comets is contained within the first twelve pages and Whipple's remarkable ' icy conglomerate ' hypothesis within one paragraph. The succeeding chapters contain the main core of the book and give an excellent introduction to cometary orbits, statistics and perturbations. The same emphasis is maintained in the later chapters of the book dealing with meteor streams, orbits and velocities. Dr. Porter has never been partial to the view that any meteors have an interstellar origin, and the most recent results serve to support the emphasis which he places on their solar system and cometary associations.

The clarity of the text, which is supported by excellent diagrams, is a very great help to the understanding of this somewhat specialized subject in astronomy. In many places the author's approach is original, particularly in the chapters on perturbations and meteor orbits. Workers in the subject should find this book of inestimable value, and the appendices, which give worked examples for the calculation of an ephemeris and meteor orbits, should be a particular asset to students. The general interest and value of the book is enhanced by some excellent photographs of comets.

A. C. B. L.

Vorlesungen über theoretische Physik : *Band IV—Optik*, by A. SOMMERFELD. Pp. xv + 389. (Wiesbaden : Dietrische Verlagsbuchhandlung, 1950.) In paper DM 6.50, in cloth DM 18.50.

The present series is probably the last in which one author will attempt to cover the whole field of theoretical physics, and Sommerfeld was probably the last of the physicists capable of accomplishing the task. Among those who attended his lectures are to be found so many in the front rank of contemporary physics that the quality of his teaching clearly matched that of his own research and that which he inspired. The fact is equally evident from the content and style of the ' Vorlesungen ' as printed.

The scope of this fourth volume is that generally understood by classical geometrical and physical optics—although the scope and level of treatment are far in advance of what is normally found under these headings.

The exposition, especially the mathematical arguments, has clearly been through the refining fire of lecturing. Only very occasionally can we feel there is a shorter way or one of greater simplicity or clarity. The evaluation of the statistical mean value of

$$\left|\sum_{n=1}^{N} \exp\{-i\boldsymbol{k}(\alpha x_n + \beta y_n)\}\right|^2$$

on pp. 195–6 is an exception which one might diffidently criticize.

This book is an essential possession for anyone working in the field of optics. It has a very wide range, and is of a standard which cannot fail to inspire admiration of one who was a great physicist and a gifted teacher.

H. H. HOPKINS.

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ABSTRACTS FOR SECTION A

The Vibrational Overlap Integral and the Intensities in Band Systems of Diatomic Molecules, by TA-YOU WU.

ABSTRACT. For the evaluation of the so-called vibrational overlap integral in the theory of the relative intensities in a band system of a diatomic molecule various approximate methods have been proposed for obtaining the vibrational wave functions. The 'distorted wave' method of Gaydon, Pearse and Pillow is here discussed. For the case of Morse oscillators an approximate method is suggested that enables the integrals to be evaluated semi-analytically. The method is applied to the calculation of the relative intensities of the second negative bands of O_2^+ studied recently experimentally by Feast. A procedure for applying the WKB method to find the vibrational wave functions of an anharmonic oscillator is also given. A general exact relation involving the vibrational wave functions of two electronic states and their potentials is given which may be used as a check for the accuracy of the approximate wave functions obtained by various methods.

On Jastrow's Interaction and the Elastic Scattering of High Energy Protons by Deuterons, by B. H. BRANSDEN.

ABSTRACT. An approximate method has been used to calculate the elastic total and differential cross sections for p–d scattering, at incident proton energies of 150 and 240 MEV. A nuclear interaction has been assumed of a type suggested by Jastrow, containing an infinite repulsive core in singlet two-body states. Of the various phenomenological interactions, this agrees about the best with the high energy n–p and p–p scattering data. The calculated angular distribution near 90°, in the centre-of-mass system, is found to be of the right order of magnitude, but agreement with experiment in the forward and backward directions is poor. An explanation of this result is suggested.

Coherent Scattering of Gamma-Rays by Bound Electrons, by G. E. BROWN and J. B. WOODWARD.

ABSTRACT. The coherent scattering of high energy gamma-rays by bound electrons in light atoms is investigated. The bound intermediate states of the electron are expanded in terms of free states and potential scatterings. It is found that not only the zero-order term of this expansion, but also the first-order term, must be kept, and that the latter is actually the larger of the two for large momentum transfer.

Angular Distribution of γ-Radiation following a Deuteron Stripping Reaction, by G. R. SATCHELOR and J. A. SPIERS.

ABSTRACT. Expressions are derived theoretically for the angular distribution of γ-radiation from a deuteron stripping reaction, measured in coincidence with a fixed direction of the emitted particle.

Some Absolute Determinations of Proton–Gamma Resonances in the Energy Range below 500 *keV*, by S. E. HUNT.

ABSTRACT. An electrostatic analyser of high accuracy has been used to measure and control the energy of a proton beam accelerated by an electrostatic generator. By using a deflecting angle of $\pi/2\sqrt{2}$ radians, a wide incident beam was focused on to a narrow exit slit.

Target currents of 10 μA were obtained homogeneous in energy to one part in a thousand. The beam energies were also measured absolutely to this accuracy.

The instrument was used to determine the energy of the ^{7}Li(p, γ) resonance, which was found to be $441{\cdot}5 \pm 0{\cdot}5$ kev with a half width of $12{\cdot}2 \pm 0{\cdot}5$ kev. The energies of two sharp resonances in fluorine were found to be $340{\cdot}4 \pm 0{\cdot}4$ and $483{\cdot}1 \pm 0{\cdot}5$ kev, with half widths $2{\cdot}9 \pm 0{\cdot}2$ kev and $2{\cdot}2 \pm 0{\cdot}2$ kev respectively.

Neutron Total Cross Sections, by E. R. HODGSON, J. F. GALLAGHER and E. M. BOWEY.

ABSTRACT. The neutron total cross sections of Be, Na, Mo and Hg have been measured in the energy region 1 ev to 10 kev, and of Th between 1 ev and 1 kev, using the Harwell linear accelerator time-of-flight spectrometer. The energy resolution is better than in previously reported work on these elements, and several resonances have been more clearly resolved.

Nuclear Disintegrations by Slow Negative μ-Mesons and Photons, by J. M. B. LANG.

ABSTRACT. The emission of particles from a nucleus following the capture of a negative μ-meson is discussed using evaporation theory. Assuming a reaction of the form $\mu^- + P \rightarrow N + \mu^0$, it is possible to calculate a distribution of nuclear excitation from which the mean probability of charged particle emission is evaluated.

In the relation between excitation U and nuclear temperature τ the exponent of τ is given the values 2 and 3. The usual law $U = \lambda\tau^2$ is found to predict results for proton emission which are far less than those experimentally observed.

In order to investigate this energy–temperature relationship data from low energy photo-disintegration experiments ($U \simeq 20$ MEV) have been compared with theoretical values. Combination of the limited evidence available with very high energy data suggests a variation of excitation with the cube of the nuclear temperature.

The Mass Difference of Sulphur 32 *and Chlorine* 35, by A. FOLKIERSKI.

ABSTRACT. The Q-values of the reactions $^{32}S(\alpha, p)^{35}Cl$ and $^{35}Cl(n, \alpha)^{32}P$ were measured as $-2{\cdot}02 \pm 0{\cdot}11$ MEV and $+1{\cdot}07 \pm 0{\cdot}15$ MEV respectively. From these Q-values and from the β-decay energy of ^{32}P the difference in mass excess of ^{32}S and ^{35}Cl was calculated in two independent ways, giving $1{\cdot}96 \pm 0{\cdot}11$ MEV and $1{\cdot}98 \pm 0{\cdot}15$ MEV.

In the investigation of the (α, p) reaction the nuclear emulsion technique was used in conjunction with α-particles from the ThC′. An ionization chamber was used to measure the α-particle energy in the case of the (n, α) reaction which was excited by fast neutrons obtained from the D–D process.

The M_{IV} and M_V Absorption Edges of ^{59}Pr, ^{60}Nd, ^{62}Sm and ^{63}Eu, by H. F. ZANDY.

ABSTRACT. Investigations on the M_{IV} and M_V absorption edges of four of the ‘ light ’ rare earth elements are described, using a bent crystal focusing spectrometer. In particular, close attention has been paid to the occurrence of fine structure (‘ white lines ’) on the long wavelength side of the edge. Such white lines are found at both edges for ^{59}Pr, ^{60}Nd, and ^{62}Sm, but for ^{63}Eu, while a white line is found at the M_V edge, it is absent ort very weak at the M_{IV} edge.

The Second Virial Coefficient near Absolute Zero, by H. S. GREEN.

ABSTRACT. By the solution of Bloch's equation for the density matrix, a development is obtained suitable for the exact evaluation of the second virial coefficient near absolute zero. Assuming a molecular interaction of the Lennard-Jones type, it is shown that the virial coefficient for helium is $-\{\frac{1}{2}(\pi\beta)^{3/2} + 31{\cdot}15\,\pi\beta + O(1)\}\text{Å}^3$, $\pi\beta = 38{\cdot}04/T$.

General formulae are given for fluids the molecules of which satisfy Bose or Fermi statistics.

The Thermal Conductivity of some Polycrystalline Solids at Low Temperatures, by R. Berman.

ABSTRACT. The thermal conductivities of polycrystalline alumina, beryllia and graphite have been measured between 2° K and room temperature. The first two are dielectric solids but, even in the graphite specimens, the electronic contribution to the heat conduction is very small.

It is shown that the phonon mean free path is limited by scattering at the crystallite boundaries at sufficiently low temperatures. For sintered specimens of alumina and beryllia, in which the density is near the single crystal value, it appears that the mean free path can become greater than the crystallite size. On the other hand there is no evidence that the mean free path can increase above the crystallite size in the graphite specimens, although in this case the interpretation of the results is more difficult.

As a consequence of this restriction on the mean free path, graphite specimens with small crystallites are extremely good heat insulators at low temperatures, being better than a glass at temperatures below 5 to 10° K.

A Note on Oriented Crystalline Overgrowth, by M. Blackman.

ABSTRACT. A brief review is given of experimental results on the oriented overgrowth of very thin deposits of alkali halides evaporated on to cleavage surfaces. The theoretical implications of some of these results are considered.

The Velocity of Second Sound in Various Media, by R. B. Dingle.

ABSTRACT. On the assumption that second sound can be propagated through media other than helium II, its velocity is calculated for non-ideal gases, insulators, conductors, superconductors and ferromagnets.

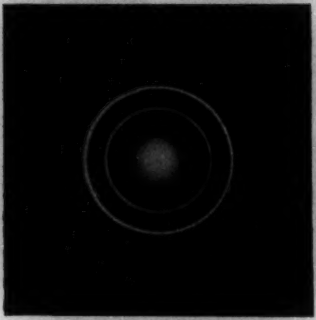

Fig. 1. Transmission tin foil.

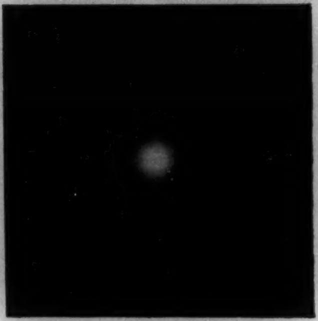

Fig. 2. Amorphous oxide pattern.

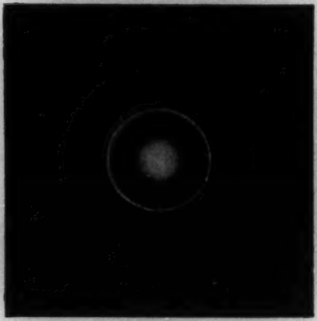

Fig. 3. SnO.

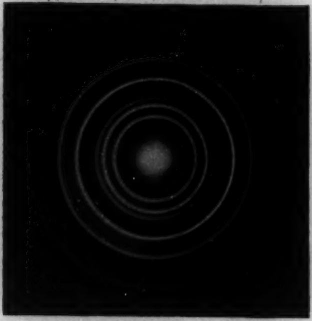

Fig. 4. SnO_2.

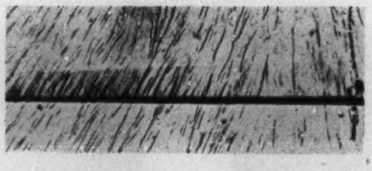

Fig. 2. Twin (dark band) with accommodation kink (×42).

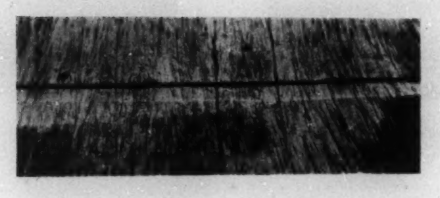

Fig. 4. Twin (dark band) with two accommodation kinks which vary in width (×42).

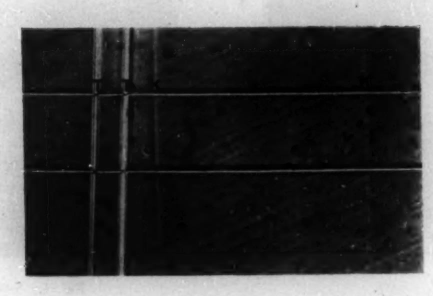

Fig. 7. Two very narrow twins (T) with two accommodation kinks (K) (×165).

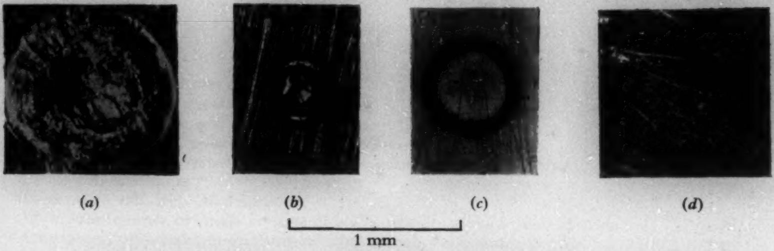
(*a*) (*b*) (*c*) (*d*)

Fig. 1. Typical anode marks. (*a*) Tin anode, arc duration 1200 μsec, arc current 34 amp. (*b*) Aluminium, 200 μsec, 50 amp. (*c*) Copper, 200 μsec, 50 amp. (*d*) Tin, 1 μsec, 100 amp.

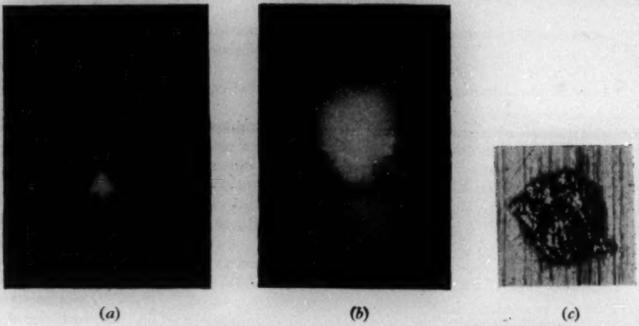
(*a*) (*b*) (*c*)

Fig. 3. Oblique photographs of the arc column and cathode spot for a 50 μsec 80 amp arc, (*a*) 6 μsec, (*b*) 40 μsec after initiation. Exposure 3 μsec. (*c*) Photomicrograph to the same scale of the cathode mark left by the arc in (*b*).

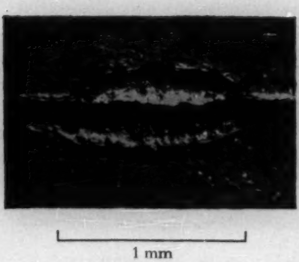

Fig. 2. Depth of melting of tin anode for a 200 μsec 50 amp arc.

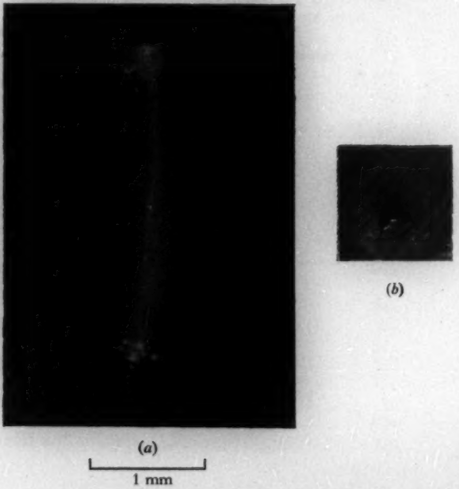
(*a*) (*b*)

Fig. 4. (*a*) 5 μsec arc to plane tin anode. (*b*) Photomicrograph of corresponding anode mark.

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